UH - Math 6302 - Dr. Heier - Fall 2015
HW 1
Due Monday, Sep. 21, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let G be a group and $H \subset G$ a finite non-empty subset. Prove that H is a subgroup of G if and only if $\forall\ x, y \in H : xy \in H$.

2. (1 point) Let G be a finite cyclic group of order k. Prove that G is isomorphic to $\mathbb{Z}_k$.

3. (1 point) Draw the "lattice of subgroups" for the symmetric group S_3.

4. (1 point) Let $\varphi : G \to H$ be a homomorphism of groups. Prove that $\ker \varphi$ is a normal subgroup of G. Prove that $\operatorname{im} \varphi$ is a subgroup of H. Is $\operatorname{im} \varphi$ always normal? Prove your answer.

5. (1 point) Section 2.1 Problem 6

6. (2 points) Section 2.2 Problem 6

7. (3 points) Section 3.2, Problem 9