Due Thursday, 09/19, at the beginning of class.
Solutions may be handwritten. Use regular sheets of paper, stapled together.
Do not forget to write your name on page 1.

1. (2 points) Prove the following statement: $\exists x, y, z \in \mathbb{Z} : (x = z^3 \wedge x + 1 = y^2)$

2. (2 points) Prove that there are 100 consecutive integers which are not perfect squares. Is your proof constructive or non-constructive?

3. (2 points) Let x be a rational number and y be an irrational number. Prove that $x + y$ is irrational.

4. Let A, B be sets. Prove that

(a) (1 point) $A \cap (B \setminus A) = \emptyset$

(b) (1 point) $A \cup (B \setminus A) = A \cup B$

5. Determine if each of these functions is a bijection $\mathbb{R} \to \mathbb{R}$.

(a) (0.5 points) $f(x) = 2x + 1$

(b) (0.5 points) $f(x) = x^2 + 1$

(c) (0.5 points) $f(x) = x^3 + 1$

(d) (0.5 points) $f(x) = \frac{x}{x^2+1}$