UH - Math 4378/6309 - Dr. Heier - Spring 2011
HW 1
Due 1/26, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(a_1, a_2, a_3) = (a_1 + 2a_2, -2a_1 + 4a_2 + a_3, a_2 + 3a_3)$. Let $\beta = \{(1,0,1),(0,2,2),(1,2,0)\}$ and $\gamma = \{(5,9,13),(1,-4,-6),(4,7,1)\}$. Compute $[T]_\beta^\gamma$.

2. (1 point) Find elementary matrices $E_1, \ldots, E_k$ with the following property. Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}.$$

Then $E_k \cdot \ldots \cdot E_1 \cdot A = I_3$. ($I_3$ denotes the three by three identity matrix.)

3. (1 point) Prove that for

$$B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix},$$

no $E_1, \ldots, E_k$ with $E_k \cdot \ldots \cdot E_1 \cdot B = I_3$ exist.

4. (1 point) Find the characteristic polynomial of

$$A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}.$$

Factor it into powers of linear factors and determine the eigenvalues (with multiplicity).

5. (1 point) Find the characteristic polynomial of

$$A = \begin{pmatrix} 8 & 5 & 6 & 0 \\ 0 & -2 & 0 & 0 \\ -10 & -5 & -8 & 0 \\ 2 & 1 & 1 & 2 \end{pmatrix}.$$

Factor it into powers of linear factors and determine the eigenvalues (with multiplicity).

6. (1 point) Prove that the matrix $A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$ is diagonalizable for all $a \in \mathbb{R}$.

7. (1 point) Find all $a \in \mathbb{R}$ such that the matrix $B = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ is diagonalizable.

8. (1 point) Prove that a square matrix A is invertible if and only if 0 is not an eigenvalue of A.

9. (1 point) Prove that if $A \in Mat_{n\times n}(\mathbb{R})$ is diagonalizable and has precisely one eigenvalue c, then $A = cI_n$.

10. (1 point) Let $A, B \in Mat_{n\times n}(\mathbb{R})$. Let $T \in Mat_{n\times n}(\mathbb{R})$ be an invertible matrix such that $B = T^{-1}AT$. Prove that A and B have the same characteristic polynomial.