UH - Math 3330 - Dr. Heier - Sample Midterm Exam - Spring 2013
Time: 75 min

1. Let the function $f : \mathbb{Z} \to \mathbb{Z}$ be defined by

$$f(x) = \begin{cases} \frac{x+1}{2} & \text{if } x \text{ is odd} \\ 2x+1 & \text{if } x \text{ is even} \end{cases}.$$

(a) (10 points) Is f one-to-one? Does f possess a left inverse? If yes, find a left inverse.

(b) (10 points) Is f onto? Does f possess a right inverse? If yes, find a right inverse.

2. (a) (10 points) Find the gcd of $a = 124$ and $b = 52$ by using the Euclidean algorithm as discussed in class. Also, find m, n such that $gcd(a, b) = ma + nb$.
(b) (10 points) Let c, d, q, r be positive integers. If $c = dq + r$, prove that $\gcd(c, d) = \gcd(d, r)$.

3. (a) (10 points) Find all solutions of the system

$$x \equiv 1 \pmod{5}$$
$$x \equiv 2 \pmod{11}.$$

(b) (10 points) Find the solution of the following system of equations in $\mathbb{Z}_7$:

$$[2][x] + [3][y] = [1]$$
$$[3][x] + [2][y] = [2].$$

4. (a) (10 points) Let $(G, *)$ be a group. Give a definition for a non-empty subset H of G to be a subgroup.
(b) (10 points) Let $\phi : G \to G'$ be a homomorphism. Let H be a subgroup of G. Prove that $\phi(H)$ is a subgroup of G'.

5. (20 points) Let G be an abelian group. Prove that the set of all elements of finite order in G forms a subgroup of G.
Remark: This subgroup is called the torsion subgroup of G. Recall that the order of a group element x is the smallest positive integer k such that $x^k = e$. If no such k exists for a given x, we say the order of x is infinite.