UH - Math 6303 - Dr. Heier - Spring 2014
HW 3
Due 03/18, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Prove that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic.

2. (2 points) Let $F \subset E \subset K$ be fields. Let E/F and K/E be Galois. Is it necessarily true that K/F is Galois? Prove your answer.

3. (1 point) Let $f(x) \in \mathbb{Q}[x]$ be a separable polynomial of degree $d \geq 3$. Is it possible that the Galois group of the splitting field of $f(x)$ is $\mathbb{Z}_2$? Prove your answer.

4. (2 points) Let $f(x) \in \mathbb{Q}[x]$ be an irreducible polynomial of degree p, where p is a prime. Assume that $f(x)$ has precisely two nonreal roots in the complex numbers. Prove that the Galois group of the splitting field of $f(x)$ is the full symmetric group S_p.

5. (2 points) Let K be the splitting field over F of a separable polynomial. Prove that if $\mathrm{Gal}(K/F)$ is cyclic, then for each divisor d of $[K : F]$ there is exactly one field E with $F \subset E \subset K$ auch that $[E : F] = d$. (Hint: Use the Fundamental Theorem of Galois Theory.)

6. (2 points) Suppose K/F is a Galois extension of degree p^n for some prime p and positive integer n. Prove that there are Galois extensions of F contained in K of degrees p and p^{n-1}. (Hint: Use the Fundamental Theorem of Galois Theory.)