UH - Math 6303 - Dr. Heier - Spring 2014
HW 5
Due no later than 04/30, 11:15am, at my office PGH 666 (if I am not in, please slide your solution under my office door).

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $\varphi : V \to W$ be a surjective morphism of affine algebraic sets. Prove that if V is an affine variety, then W is an affine variety.

2. (1 point) Let $V \subseteq \mathbb{A}^n$ be an affine variety and $f \in k[V]$. Prove that the graph of f is an affine variety.

3. (1 point) Prove that $GL_n(k)$ is a Zariski-open subset of $\mathbb{A}^{n^2}$ and can be embedded as an affine algebraic set in $\mathbb{A}^{n^2+1}$.

4. (2 points) Prove that if k is infinite, the set $\{(a, a^2, a^3) | a \in k\} \subset \mathbb{A}^3$ is an affine variety.

5. (1 point) Give an example of an integral domain which is not normal. You don't need to prove that your ring is an integral domain, but you do need to prove it is not normal.

6. (1 point per item)

(a) Let V be an affine algebraic set in $\mathbb{A}^n_{\mathbb{R}}$. Prove that there is a polynomial $f \in \mathbb{R}[x_1, \ldots, x_n]$ such that $V = \mathcal{Z}(f)$.

(b) Does the same hold with $\mathbb{R}$ replaced by $\mathbb{C}$? Prove your answer.

7. (2 points) Let k be an algebraically closed field. Use Hilbert's Nullstellensatz to prove that every proper radical ideal in $k[x_1, \ldots, x_n]$ is the intersection of maximal ideals.