UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 1
Due Friday, 01/27, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let S, T be sets. We define the *set-theoretic difference* of the ordered pair (S, T) to be

$$S \setminus T = \{x \in S \,|\, x \notin T\}.$$

(a) (1 point) Prove that $T \cup (S \setminus T) = S \cup T$.

(b) (1 point) Prove that $(S \setminus T) \cup (S \cap T) = S$.

2. Let A, B, C be sets.

(a) (1 point) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(b) (1 point) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

3. (2 points) Prove that, for all positive integers n,

$$\sum_{i=1}^{n} 2^i = 2(2^n - 1).$$

4. (2 points) Prove that, for all integers $n \geq 4$,

$$2^n < n!.$$

5. (2 points) How many binary operations are there on a set S with n elements? How many of these binary operations are commutative? Justify your answer carefully.