Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let R be a ring with unity 1. Prove that $(-1)a = -a$ for all $a \in R$.

2. An element r in a ring is called *idempotent* if $r^2 = r$. Let R be a ring with unity 1. Let $r \in R$ be idempotent. Prove that

(a) (1 point) $1 - r$ is also idempotent;

(b) (1 point) r or $1 - r$ is a zero-divisor.

3. Let R be a ring with unity. Then R is called *Boolean* if every element of R is idempotent. Prove that if R is Boolean, then

(a) (1 point) $r = -r$ for every $r \in R$;

(b) (1 point) R is commutative.

4. (2 points) Let R be a ring with unity. Assume that R has no non-zero zero-divisors. Let $a, b \in R$ with $ab = 1$. Prove that $ba = 1$.

5. (2 points) Let F be the set of all 2×2 matrices of real numbers of the form

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}.$$

Prove that F forms a field under the usual addition and multiplication of matrices.