UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 12 (final HW set for the course)
Due Friday, 04/28, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Prove that $\mathbb{Z}_n$ is a field if and only if n is a prime number.

2. (2 points) Let R be a ring with unity. Assume that for all x and y in R we have $(xy)^2 = x^2y^2$. Prove that R is commutative.

3. (2 points) Let R be a commutative ring with unity. Let $a \in R$. Prove that $aR = R$ holds if and only if a is a unit.

4. Let $S = \{q \in \mathbb{Q} : q = \frac{a}{b},\ a, b \in \mathbb{Z} \text{ and } b \text{ odd}\}$.

(a) (1 point) Prove that S is a subring of $\mathbb{Q}$.

(b) (1 point) Prove that S has a unique maximal ideal.

5. Let R be a commutative ring with unity $1 \neq 0$.

(a) (1 point) Prove that R is an integral domain if and only if $\{0\}$ is a prime ideal in R.

(b) (1 point) Prove that R is a field if and only if $\{0\}$ is a maximal ideal in R.