UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 2
Due Friday, 02/03, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Does addition yield a binary operation ...

(a) (1 point) on the set $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$ of even integers? If yes, is the set with the binary operation a group?

(b) (1 point) on the set $\{\ldots, -3, -1, 1, 3, \ldots\}$ of odd integers? If yes, is the set with the binary operation a group?

2. In class, we defined a binary operation $\oplus$ on $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$. We now define a binary operation $\odot$ on $\mathbb{Z}_n$ by setting $a \odot b := \overline{a \cdot b}$.

(a) (1 point) Prove that $\odot$ is associative.

(b) (0.5 points) Does $Z_4 \setminus \{0\}$ form a group with $\odot$? Prove your answer.

(c) (0.5 points) Does $Z_5 \setminus \{0\}$ form a group with $\odot$? Prove your answer.

3. In $\mathbb{Z}_{13}$, solve

(a) (1 point) the equation $6 \oplus 9 \oplus x \oplus 2 = 7$ for x.

(b) (1 point) the equation $7 \odot x = 5$ for x.

4. (2 points) Let $(G, *)$ be a group such that $x * x = e$ for all $x \in G$. Prove that G is abelian.

5. (2 points) Let $(G, *)$ be a group. Prove that G is abelian if and only if $(x * y)^2 = x^2 * y^2$ for all $x, y \in G$.