UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 4
Due Friday, 02/17, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let G be a group.

(a) (1 point) Prove that if $G = \langle x \rangle$, then $G = \langle x^{-1} \rangle$.

(b) (1 point) Prove that if $G = \langle x \rangle$ and G is infinite, then x and x^{-1} are the only generators of G.

2. Let H, K be subgroups of a group G.

(a) (1 point) Prove that $H \cap K$ is a subgroup of G.

(b) (1 point) Prove that $H \cup K$ is a subgroup of G if and only if ($H \subset K$ or $K \subset H$).

3. Prove that the following sets H of matrices are subgroups of $GL(2, \mathbb{R})$.

(a) (1 point) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + c = 1,\ b + d = 1,\ ad - bc \neq 0,\ a, b, c, d \in \mathbb{R} \right\}$

(b) (1 point) $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1,\ a, b \in \mathbb{R} \right\}$

4. (2 points) Let H be a nonempty finite subset of a group H. Assume that H is closed under inverses. Must H be a subgroup of G? Either give a proof or a counterexample.

5. (2 points) Let G be a group and let H be a nonempty subset of G such that whenever $x, y \in H$, we have $x(y^{-1}) \in H$. Prove that H is a subgroup of G.