UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 6
Due Friday, 03/03, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Execute the following multiplications in S_7.

(a) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 7 & 1 & 3 & 2 & 6 & 5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 1 & 5 & 3 & 2 & 4 & 7 \end{pmatrix}$.

(b) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 7 & 5 & 6 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}$.

2. Write each of the following permutations as a product of disjoint cycles and then as a product of transpositions. Determine whether each permutation is odd or even.

(a) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 3 & 9 & 6 & 7 & 8 & 5 & 10 & 1 & 2 \end{pmatrix}$.

(b) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 3 & 1 & 6 & 4 & 5 & 8 & 9 & 2 & 10 \end{pmatrix}$.

3.

(a) (1 point) Give an example of two elements x, y in S_9 such that $o(x) = o(y) = 5$ and $o(xy) = 9$.

(b) (1 point) What is the largest order an element of S_9 can have? Prove your answer.

4. (2 points) Find elements $x, y \in S_{\mathbb{Z}}$ such that x and y have finite order, yet xy has infinite order.

5.

(a) (1 point) Let G be a group and $a, b \in G$. Let $a \sim b$ hold if and only if there exists $x \in G$ such that $a = xbx^{-1}$. Prove that $\sim$ is an equivalence relation.

(b) (1 point) For integers x, y, let $x \sim y$ hold if and only if $3x - 10y$ is a multiple of 7. Prove that $\sim$ is an equivalence relation.