UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 7
Due Friday, 03/10, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1.

(a) (1 point) Let the relation $\sim$ on $\mathbb{R}$ be defined by $x \sim y$ if and only if $|x - y| < 1$. Is this an equivalence relation? Prove your answer.

(b) (1 point) Let the relation $\sim$ on $\mathbb{Z}$ be defined by $x \sim y$ if and only if $(-1)^x = (-1)^y$. Is this an equivalence relation? Prove your answer.

2.

(a) (1 point) Find the right cosets of the subgroup $H = \{(0,0),(1,0),(2,0)\}$ in $\mathbb{Z}_3 \times \mathbb{Z}_2$.

(b) (1 point) Find the right cosets of the subgroup $H = \{(0,0),(0,2)\}$ in $\mathbb{Z}_4 \times \mathbb{Z}_4$.

3. (2 points) Let p, q be two prime numbers, and let G be a group of order pq. Show that every subgroup H of G with $H \neq G$ is cyclic.

4. (2 points) Let G be a group of order p^2, where p is a prime. Prove that G must have a subgroup of order p.

5. (2 points) Let $G = \{e, x_1, \ldots, x_{r-1}\}$ be an abelian group such that $r = |G|$ is an odd integer. Prove that

$$x_1 \cdot \ldots \cdot x_{r-1} = e.$$

Hint: Prove first that $x_1 \cdot \ldots \cdot x_{r-1}$ is its own inverse.