UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 8
Due Friday, 03/31, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Let H be the subgroup of $GL(2,\mathbb{R})$ consisting of all matrices

$$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$$

with $a, d \neq 0$. Is H a normal subgroup of $GL(2,\mathbb{R})$? Prove your answer.

2. (2 points) Let G be a group, let $g \in G$ have finite order m, and let $H \lhd G$. Prove that the order of the element $Hg \in G/H$ is finite and divides m.

3. (2 points) Let p, q be two distinct prime numbers, and let G be an abelian group of order pq. Prove that G is cyclic.

4. (2 points) Prove that $(\mathbb{Q},+)/(\mathbb{Z},+)$ is an infinite group such that each of its elements has finite order.

5. A subgroup H of a group G is characteristic if $\varphi(H) \subseteq H$ for every automorphism φ of G.

(a) (1 point) Prove that every characteristic subgroup is normal.

(b) (1 point) Prove that the converse of (i) is false.