UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 9
Due Friday, 04/07, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Let G and H be groups. Prove that $G \times H$ and $H \times G$ are isomorphic groups.

2. (2 points) Let $G = \mathbb{Z}_2 \times \mathbb{Z}_4$. Find subgroups H and K of G such that H is isomorphic to K, but G/H and G/K are not isomorphic. Justify your answers carefully.

3. (2 points) If G is a group, let $Aut(G)$ denote the set of automorphisms of G. Show that $Aut(G)$ is a subgroup of $(S_G, \circ)$.

4. (2 points) Prove that the group of automorphisms of $(\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Z}_2, \oplus)$.

5. (2 points) Let H be a subgroup of G with $H \neq G$ and let ψ be an automorphism of H other than the identity mapping. Define a mapping $\varphi : G \to G$ by

$$\varphi(x) = \begin{cases} \psi(x) & \text{if } x \in H \\ x & \text{if } x \notin H \end{cases}.$$

Is φ an automorphism of G? Explain.