UH Math 3330-01 Dr.Heier-Spring 2017
HW8 Answer Key

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Problem1.
H is not a normal subgroup of $GL(2,\mathbb{R})$.

Problem2.

Proof. Let e be the identity of G, From the assumption we know that $g^m = e$. $(Hg)^m = Hg^m = He = H$ so $ord(Hg)|m$. □

Problem3.

Proof. G is an abelian group of order pq then by Cauchy's Theorem there are elements $a, b \in G$ s.t. $ord(a) = p, ord(b) = q$. Then $ord(ab) = pq = |G|$. Then G is cyclic. □

Problem4.

Proof. For every $m \neq n \in \{2, 3, 4, 5\ldots\}$ we have $\frac{1}{m} - \frac{1}{n} \notin \mathbb{Z}$ then $\{\frac{1}{n} + \mathbb{Z}\}$ is an infinite subset of $\mathbb{Q}/\mathbb{Z}$, so $\mathbb{Q}/\mathbb{Z}$ must be infinite.

Every rational number can be written as $\frac{m}{n}$ for integers m, n, thus for every element $\frac{m}{n} + \mathbb{Z}$,

$$\frac{m}{n} + \mathbb{Z} + ... + \frac{m}{n} + \mathbb{Z}(\text{n times}) = m + \mathbb{Z} = \mathbb{Z}. \tag{1}$$

Then $ord(\frac{m}{n} + \mathbb{Z})|n$, which is finite. □

Problem5.
Hint:
(a) If H is characteristic then consider the conjugate automorphism on G.
(b) False, consider counter example $\mathbb{Q}$ with automorphism $x \mapsto \frac{1}{2}x$, and normal subgroup $\mathbb{Z}$.