UH - Math 3330 - Dr. Heier - Spring 2019
HW 5
Due Wednesday, 02/20, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let $f : A \to B$ be a function between non-empty sets A, B.

(a) (1 point) Prove that for all subsets $S, T \subseteq A$, $f(S \cup T) = f(S) \cup f(T)$.

(b) (0.5 points) Prove that for all subsets $S, T \subseteq A$, $f(S \cap T) \subseteq f(S) \cap f(T)$.

(c) (0.5 points) Give an example where the containment relation in item (b) is strict.

2. Let $f : A \to B$ be a function between non-empty sets A, B.

(a) (1 point) Prove that f is injective if and only if there exists a function $g : B \to A$ such that $g \circ f = \mathrm{id}_A$, where $\mathrm{id}_A : A \to A, a \mapsto a$ is the identity function.

(b) (1 point) Prove that f is surjective if and only if there exists a function $g : B \to A$ such that $f \circ g = \mathrm{id}_B$.

3. Let $f : A \to B$ and $g : B \to C$ be functions.

(a) (1 point) Assume that $g \circ f$ is injective. Does this imply that both f and g are injective? Prove your answer.

(b) (1 point) Assume that $g \circ f$ is surjective. Does this imply that both f and g are surjective? Prove your answer.

4. Execute the following multiplications in S_7.

(a) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 7 & 1 & 3 & 2 & 6 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 1 & 3 & 5 & 7 & 4 & 2 \end{pmatrix}$.

(b) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 7 & 4 & 6 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 6 & 4 & 2 & 3 & 1 \end{pmatrix}$.

5. Write each of the following permutations as a product of disjoint cycles and then as a product of transpositions. Determine whether each permutation is odd or even.

(a) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 3 & 9 & 6 & 7 & 8 & 5 & 1 & 10 & 2 \end{pmatrix}$.

(b) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 3 & 1 & 6 & 4 & 5 & 8 & 9 & 10 & 2 \end{pmatrix}$.