UH - Math 3330 - Dr. Heier - Spring 2020
HW 1
Due Thursday, 01/23, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let S, T be sets. We define the *set-theoretic difference* of the ordered pair (S, T) to be

$$S \setminus T = \{x \in S \,|\, x \notin T\}.$$

(a) (1 point) Prove that $T \cap (S \setminus T) = \emptyset$.

(b) (1 point) Prove that $(S \setminus T) \cup (S \cap T) = S$.

2. Let A, B, C be sets.

(a) (1 point) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(b) (1 point) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

3. (2 points) Prove that, for all $n \in \mathbb{N} = \{0, 1, 2, 3, \ldots\}$,

$$\sum_{i=0}^{n} 3^i = \frac{1}{2}(3^{n+1} - 1).$$

4. (1 point) Prove that, for all integers $n \geq 5$,

$$4n < 2^n.$$

5. (2 points) The *Fibonacci sequence* f_n is defined by $f_1 = f_2 = 1$ and

$$f_n = f_{n-1} + f_{n-2}$$

for all integers $n \geq 3$. Prove that for every integer $k \geq 1$, the Fibonacci number f_{5k} is divisible by 5.

6. (1 point) Let a_n be the sequence defined by $a_1 = 1, a_2 = 8$ and for all integers $n \geq 3$:

$$a_n = a_{n-1} + 2a_{n-2}.$$

Prove that for all positive integers n,

$$a_n = 3 \cdot 2^{n-1} + 2(-1)^n$$

holds.