UH - Math 3330 - Dr. Heier - Spring 2020
HW 2
Due Thursday, 01/30, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Does addition yield a binary operation ...

(a) (1 point) on the set $\{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots\}$ of multiples of 3? If yes, is the set with the binary operation a group?

(b) (1 point) on the set $\{\ldots, -3, -1, 1, 3, \ldots\}$ of odd integers? If yes, is the set with the binary operation a group?

2. (2 points) Let G be the set of all 2×2 matrices

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix},$$

where $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Prove that G forms a group with the usual matrix multiplication. You may freely use basic facts from linear algebra without proof.

3. (1 point) Let G be a group. Let $a_1, \ldots, a_n$ be elements of G. Prove that $(a_1 \ldots a_n)^{-1} = a_n^{-1} \ldots a_1^{-1}$. You must use induction to carefully prove this statement.

4. (1 point) Let $(G, *)$ be a group such that $x * x = e$ for all $x \in G$. Prove that G is abelian.

5. In class, we defined a binary operation $\oplus$ on $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$. We now define a binary operation $\odot$ on $\mathbb{Z}_n$ by setting $a \odot b := \overline{a \cdot b}$.

(a) (1 point) Prove that $\odot$ is associative.

(b) (0.5 points) Does $Z_4 \setminus \{0\}$ form a group with $\odot$? Prove your answer.

(c) (0.5 points) Does $Z_5 \setminus \{0\}$ form a group with $\odot$? Prove your answer.

6. In $\mathbb{Z}_{13}$, solve

(a) (1 point) the equation $2 \oplus 8 \oplus x \oplus 4 = 7$ for x.

(b) (1 point) the equation $11 \odot x = 10$ for x.