UH - Math 3330 - Dr. Heier - Spring 2020
HW 6
Due Thursday, 02/27, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (1 point) Write the following permutation as a product of transpositions. Determine whether it is odd or even.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 11 & 5 & 14 & 10 & 6 & 15 & 12 & 13 & 1 & 7 & 8 & 4 & 9 & 2 & 3 \end{pmatrix}.$$

2.

(a) (1 point) Let G be a group and $a, b \in G$. Let $a \sim b$ hold if and only if there exists $x \in G$ such that $a = xbx^{-1}$. Prove that $\sim$ is an equivalence relation.

(b) (1 point) For integers x, y, let $x \sim y$ hold if and only if $11x - 3y$ is an integer multiple of 8. Prove that $\sim$ is an equivalence relation.

3. (1 point) Let n be a positive integer. Let x, y be integers. We say that x, y are congruent mod n (written $x \equiv y \mod n$) if $x - y$ is an integer multiple of n. Prove that this defines an equivalence relation on the integers.

4. (2 points) Let p, q be two prime numbers, and let G be a group of order pq. Show that every subgroup H of G with $H \neq G$ is cyclic.

5. (2 points) Let G be a group of order p^2, where p is a prime. Prove that G must have a subgroup of order p.

6. (2 points) Let G be a group. Let H, K be subgroups of G. Assume that $\#H = 12$ and $\#K = 17$. Prove that $H \cap K = \{e\}$.