UH - Math 3330 - Dr. Heier - Spring 2020
HW 7
Due Thursday, 03/05, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (1 point) Let $G = \{e, x_1, \ldots, x_{r-1}\}$ be an abelian group such that $r = \#G$ is an odd integer. Prove that

$$x_1 \cdot \ldots \cdot x_{r-1} = e.$$

Hint: Prove first that $x_1 \cdot \ldots \cdot x_{r-1}$ is its own inverse. Carefully explain your reasoning.

2.

(a) (1 point) Find the right cosets of the subgroup $H = \{(0,0),(1,0),(2,0)\}$ in $\mathbb{Z}_3 \times \mathbb{Z}_3$.

(b) (1 point) Find the right cosets of the subgroup $H = \{(0,0),(0,2)\}$ in $\mathbb{Z}_3 \times \mathbb{Z}_4$.

3. (1 point) Let H be a normal subgroup of G with $\#H = 2$. Prove that $H \subset Z(G)$.

4. (2 points) Let G be a group and let H, K be two normal subgroups of G with $H \cap K = \{e\}$. Prove that for $x \in H$ and $y \in K$, $xy = yx$ holds.

5. (2 points) Let G be a group and let N a normal subgroup of G. Let H be a subgroup of G. Set $NH = \{nh \mid n \in N,\ h \in H\}$. Prove that NH is a subgroup of G.

6. (2 points) Let G be a group and let H a normal subgroup of G such that $[G : H] = 20$ and $\#H = 7$. Suppose $x \in G$ and $x^7 = e$. Prove that $x \in H$.