UH - Math 4377/6308 - Dr. Heier - Spring 2020
HW 1
Due date: 01/23, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $A = \{1, 3, 5, 7, 8\}$, $B = \{4, 5, 7\}$, $C = \{4, 6, 7\}$. Explicitly write down the sets

$$A \cup B \cup C,\ A \cap B \cap C,\ A \cap (B \cup C),\ B \setminus (A \cup C),\ B \setminus (A \cap C),\ A \times B.$$

2. Let $x, y \in \mathbb{Z}$. Prove or disprove that the following relations are equivalence relations.

(a) (0.5 points) $x \sim y$ if and only if $x - y$ is greater than -1.

(b) (0.5 points) $x \sim y$ if and only if $x \cdot y \leq 0$.

(c) (0.5 points) $x \sim y$ if and only if $y + 7x$ is an integer multiple of 8.

3. (1 point) Let $f : A \to B$ and $g : B \to C$ be functions. Assume that f is injective and that $g \circ f$ is injective. Does this imply that g is injective? Prove your answer.

4. Let the function $f : \mathbb{Z} \to \mathbb{Z}$ be defined by

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \text{ is even} \\ 3x + 1 & \text{if } x \text{ is odd} \end{cases}.$$

(a) (1 point) Is f injective? Prove your answer.

(b) (1 point) Is f surjective? Prove your answer.

5. (1 point) Prove carefully that in any field F, all $a, b \in F$ satisfy $(-a) \cdot (-b) = a \cdot b$. Here, for any $x \in F$, $-x$ denotes the unique additive inverse of x.

6. (1.5 points) Prove that the set of numbers $\{x + y\sqrt{5} \,|\, x, y \in \mathbb{Q}\}$ is a field with the usual addition and multiplication of reals.

7. (1 point) Let $z = 1 + 3i$, $w = 1 - i$. Write $\bar{w}$, $3z - 2w$, $z\bar{w}$, $|\bar{z}|$, $\frac{w}{z}$ in the form $a + bi$.

8. (1 point) Find all solutions of the equation $z^2 - 4z + 8 = 0$ in $\mathbb{C}$.