On a conjecture by C. Sundberg: A numerical investigation

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Abstract

Carl Sundberg (University of Tennessee-Knoxville) conjectured some time ago that

$$\sup_{\varphi \in E} \frac{\displaystyle\int_0^1 \frac{|\varphi'|^4}{\varphi^6}\,dx}{1+\displaystyle\int_0^1 |\varphi''|^2\,dx} < +\infty, \qquad \text{(SI)}$$

where

$$E = \{\varphi \mid \varphi \in H^2(0, 1),\ \varphi(0) = \varphi(1),\ \varphi'(0) = \varphi'(1),\ \varphi \geq 1\}.$$

Our goal in this lecture is to report on the results of a numerical investigation that has been carried out these last few months in order to verify the veracity of the above Sundberg inequality. Indeed, our numerical experiments strongly suggest that (SI) is true and give also an approximation of the numerical value of the supremum over E of the functional in (SI). A brief description of the numerical methodology used to verify (SI) will be also provided.

Since the above maximization problem has some of the features of an ***obstacle problem*** and that the methodology we employ relies on a well-chosen ***augmented Lagrangian*** functional, it is our opinion that the problem we address in this presentation is relevant to the many scientific contributions of ***R. Hoppe***.