# The Putnam Competition from 1938-2012*

Joseph A. Gallian

**1. INTRODUCTION**. The William Lowell Putnam Competition is held annually for the top undergraduate mathematics students in the United States and Canada. The first Putnam competition took place in 1938, but its genesis was a math competition held in 1933 between ten Harvard students and ten students from the United States Military Academy at West Point [2]. That competition was sponsored by Elizabeth Lowell Putnam in honor of her late husband William Lowell Putnam, who was a member of the Harvard class of 1882. That competition went so well that plans were made to have an annual competition in which all interested institutions could participate. This came about in 1938, when the first official Putnam competition was sponsored by the Mathematical Association of America. The examination was prepared and graded by members of the Harvard mathematics department and Harvard students were excluded the first year. There were both individual and team competitions. The questions were drawn from calculus, the theory of equations, differential equations, and geometry. (The problems are included at the end of this article.) Prizes in the first few years were \$500, \$300, and \$200 for the top three teams and \$50 each for the top five ranking individuals, who were designated as Putnam Fellows. By the year 1997 the prizes for the top five teams were \$25,000, \$20,000, \$15,000, \$10,000, and \$5,000, while Putnam Fellows received \$2,500 each. Moreover, each year one Putnam Fellow receives the William Lowell Putnam Fellowship for graduate study at Harvard.

The first competition had 163 individuals and 42 teams. The number of participants exceeded 1,000 for the first time in 1961, when 1,094 individuals and 165 teams took part. In 2011 there were 4440 students representing 572 institutions and 460 teams. All three of these totals are the highest on record. The number of participants in the 2011 competition alone exceeds the total number of participants in the first 19 competitions from 1938 through the fall of 1958. (The competitions were suspended from 1943-1945 because of World War II; in 1958 there were two competitions–one in the spring and one in the fall.) Coincidentally, in both 1980 and 1981 there were exactly 2,043 participants. Through 2012, there have been 131,881 participants. The 1946 contest, coming right after the war, had the lowest participation ever with just 67 contestants and 14 teams. In 2011 there were 163 participants from MIT alone. That exceeds the total number of participants in each of 1941, 1942, 1946, 1947, 1948, and 1949. Table 1 at the end of this article provides the list of the number of participants in each of the seventy-three competitions through 2012.

In the first twenty-two competitions the number of questions varied from eleven to fourteen, but beginning with the 23rd competition in 1962, the exams have consisted of a three-hour morning session and a three-hour afternoon session, each having six questions worth ten points apiece. Institutions entering teams must designate the three team members before the competition is held. The team score is the sum of the ranks of the three team members. Thus, a team whose members finish in twenty-first, forty-ninth, and one hundred and second places has a score of 172. The lower a team's score, the higher its ranking. This method of team scoring places great weight on the lowest scoring member of the team since there is much bunching at lower scores. For example, in 1988 a team member with a score of ten ranked 1496, but a team member with a score of nine ranked 1686. In 2006 a score of one point generated 1266.5 team points, whereas a score of zero on that exam resulted in 2501 team points. Thus, even a one point difference in an individual's score can mean over a thousand points more for the team.

The fact that the team members are designated in advance and the method of summing the ranks

*This is an updated version of an article published in the American Mathematical Monthly [5] in 2004.

for team scoring causes some peculiar results on occasion. In 1959, for instance, Harvard had four Putnam Fellows but finished fourth in the team competition, and in 1966, 1970, 2005 and 2006 MIT had three Putnam Fellows but did not win the competition. There have been sixteen competitions in which the winning institution did not have a Putnam Fellow.

One might wonder about the most difficult Putnam problems over the years. By design, the 5th and 6th problems in each session tend to be more difficult than the others but time also is a factor with the last two problems. Using data from 1974-2012, the only problems for which no one in the top 200 received a positive score were A6 on the 1979 exam and B6 on the 2011 exam. These two problems are reproduced in the Appendix II. In 1999 for both B4 and B5 only a single person in the top 200 (approximately) received a positive score. In each instance the score was two. In 1980, there was only one person among the top 200 who received an 8 or more on problems B5 and one person who received a score or 8 or more on B6. In 1982 no one in the top 200 scored a 7 or more on A6. In 1993 only one person in the top 200 scored an 8 or more on A6 and only 2 received an 8 or more on B6. In 2011 only one person in the top 200 scored a positive score on A5 and B6 combined. In 2011 181 people out of the top 200 left A5 blank. There are a few anomalies such as in 1988 when 59 of the top 200 received 10 points for A6 and 38 received 10 points on B6. The strangest year was 1974 when only one person in the top 200 received 9 or 10 on B1 and only one received 9 or 10 on B2 but 34 received 9 or 10 on B6.

**2. TEAM PERFORMANCE**. By a wide margin, Harvard has the best record in the Putnam competition. Through 2012, Harvard has won the team competition twenty-nine times, while its closest rival for team titles, Caltech, has won the team title ten times. MIT is in third place with six titles with three of these coming since 2003. Tied for fourth place with four team titles each are Washington University and the University of Toronto. All four of Toronto's team titles occurred in the first six years of the competition. Toronto might have won all of the first six competitions except for the fact that it chose to disqualify itself in 1939 and 1941 because the Toronto mathematics department had prepared the questions. Starting with the fifth competition the questions have been prepared by a committee selected from different schools rather than having the department of the winning team of the previous competition prepare them. This meant that the winner of the previous year would not have to disqualify itself. Curiously, the Harvard team did not place in the top five in the first six competitions, but it has placed in the top five in fifty-seven of the seventy-three competitions held through 2012. During the first twenty competitions (1938-1959), the New York institutions Brooklyn College, Polytechnic Institute of Brooklyn, Columbia University, and City College of New York excelled in the team competition and in producing Putnam Fellows. Caltech's glory years were the six years 1971-1976 when they won the team competition five times. Excluding Harvard, only once has the same institution won three years in a row. That was Caltech in 1971-1973. Between 1976 and 1986 Washington University won the team title four times and placed second four times. During that period Wash U had only two Putnam Fellows. Beginning about 1990 Duke University started to recruit the nation's best high school math students with the same fervor that they recruit the best high school basketball players. Between 1990 and 2000 Duke became Harvard's top rival by winning three times and finishing second to Harvard twice. With these accomplishments together with its third place finish each year from 2001 to 2005, Duke's Putnam team performed as well as its men's basketball team! After finishing in the top five twenty-four times and in second place nine times prior to 2006, Princeton won its first team title in 2006. Over the years Duke won the team championship three times while having only a total of six Putnam Fellows whereas Princeton won one team championship despite having 21 Putnam Fellows. The only state universities in the U. S. to win the team competition are Michigan State (three times), and the Universities of California at Davis (once) and at Berkeley (once). The highest place ever achieved by a liberal arts college was second by Oberlin College in 1972. That same year Swarthmore finished fourth. Harvard's longest winning streak was eight years (1985-1992), and its longest stretch without winning was fifteen years (1967-1981). The only tie for first place occurred in 1984 between the University of California at Davis and Washington University. Amazingly, in 1986, 1987, and 1990 every member of Harvard's

team was a Putnam Fellow.

A complete list of the top five schools and top five individuals each year can be found at http://en.wikipedia.org/wiki/Putnam_competition. Table 3 following lists every team that has placed fifth or higher in at least one competition along with the total number of Putnam Fellows from each of these institutions. The last four entries in the table list the institutions that have not placed in the top five in the team competition but have had at least two Putnam Fellows.

**3. INDIVIDUAL ACCOLADES.** As for producing Putnam Fellows, Harvard is again the overwhelming winner with 102 versus MIT's second place fifty-six. On the other hand, between 2001 and 2012, MIT out did Harvard in Putnam Fellows twenty-five to fifteen. Harvard has had four Putnam Fellows in the same competition on four occasions. Oddly, Harvard did not record its first Putnam Fellow until the sixth competition. Since then the longest period in which Harvard did not have a Putnam Fellow is three years and that happened only once. With the exception of 2004, Harvard has had a Putnam Fellow every year since 1990. Because of tie scores for fourth or fifth place, in fourteen competitions there have been six Putnam Fellows, while in 1959 a four-way tie for fifth place resulted in eight. Thirteen of the fifteen competitions in which there were more than five Putnam Fellows have occurred since 1970. Through 2012, there have been 278 individuals who have been Putnam Fellows for a total of 382, counting multiplicity. Only eight people–Don Coppersmith, Arthur Rubin, Bjorn Poonen, Ravi Vakil, Gabriel Carroll, Reid Barton, Daniel Kane, and Brian Lawrence–have been Putnam Fellows four times. Nineteen people have been three-time winners: Andrew Gleason, Edward Kaplan, Donald J. Newman, James Herreshoff, Samuel Klein, Randall Dougherty, Eric Carlson, David Ash, Noam Elkies, David Moews, David Grabiner, Kiran Kedlaya, Lenny Ng, J. P. Grossman, Ciprian Manolescu, Aaron Pixton, Arnav Tripathy, Yufei Zhao, and Xiaosheng Mu.[1] Zhao missed being a four time Fellow by one point in 2007. In Ash's fourth attempt at the Putnam in 1984 he finished tied for sixth, just two points short of being a Putnam Fellow again. It should be noted that some of the three-time winners only took the exam three times. Barton is the only person ever to win four gold medals in four attempts in the International Mathematical Olympiad for high school students. Through 2012 there have been forty-three people who have been Putnam Fellows exactly twice. It appears that there have never been two members of the same immediate family who have been Putnam Fellows. The closest are brothers Doug and Irwin Jungreis. Doug finished in the top five in 1985 and 1986 and Irwin finished in the second five in 1980 and 1982. The first certain occurrence of a woman finishing in the Honorable Mention or higher categories was in 1948. In the announcement in the American Mathematical Monthly [7] she is listed as "M. Djorup (Miss), Ursinus College." Because many participants use the initials of their first and middle names (e.g., R. P. Feynman) it is possible that Djorup is not the first woman to achieve Honorable Mention or better status. The first woman Putnam Fellow was Ioana Dumitriu from New York University in 1996; the second was Melanie Wood from Duke in 2002; the third was Ana Caraiani from Princeton in 2003 and 2004. Since the ages of participants are not noted, there is no way to know who the youngest and oldest people to win the competition were. Most likely the youngest is Arthur Rubin, who was a winner in 1970 at age 14. John Tillinghast, David Ash, Noam Elkies and Lenny Ng were Putnam Fellows at sixteen.[2] A potential oldest winner is Samuel Klein, who was born in 1934 and won the competitions in 1953, 1959, and 1960. As a group, the five winners of the 2003 competition have amassed the greatest number of Putnam Fellow designations ever: Gabriel Carroll, Reid Barton and Daniel Kane won four time, Ana Caraiani won twice, and Ralph Furmaniak won once.

Unlike the early years of the Putnam competition, in the past twenty-five years or so many of those who have done exceptionally well in the Putnam competition have participated as high-school students in problem solving summer training camps in the United States and elsewhere in preparation for the annual International Mathematical Olympiad (IMO). In recent years, many of the international students who represented their countries in the IMO have come to the United States for their undergraduate degrees. The consequence is that the winners of Putnam competitions now come from many countries.

[1] The MAA should create action figures for all the people who were Putnam Fellows three or more times.

[2] In the version of this article published in the Monthly I had Elkies as the youngest winner that I knew of.

The 2006 Putnam competition illustrates this well. All five 2006 Putnam winners were IMO gold medal recipients and 12 of the top 26 scorers in competition represented countries other than the United States or Canada in the IMO. In 2007 five of the six Putnam Fellows were IMO Gold medalists and nine of the top 24 in the Putnam competition represented countries other than the United States or Canada in the IMO. In 2009 seven of the top 25 in the Putnam competition represented countries other than the United States or Canada in the IMO while in 2010 there were six in the top 25. In 2010 and 2012 all five Putnam Fellows were IMO Gold medalists and as were four of the five in 2011.

Over the seventy-three competitions between 1938 and 2012 there have been only four perfect scores– one in 1987, two in 1988, and one in 2010. Although the top five scorers are always listed alphabetically, it is known that the 1987 perfect score was achieved by David Moews. What is amazing about this score is that the 1987 exam was a difficult one. The median score was one point and twenty-six points put one in the top two hundred (out of 2,170 participants). In 1987 the second highest score was 108, the third highest score in 1988 was 119, and the second highest in 2010 was 118. The winners of the 1987 and 1988 competitions rank among the strongest groups of Putnam Fellows ever. Among them are Bjorn Poonen and Ravi Vakil, both four-time Putnam Fellows, David Moews and David Grabiner, both three-time Putnam Fellows, and Mike Reid, a two-time Putnam Fellow. In contrast to the 1988 scores, of the 1,260 contestants in the 1963 competition the highest score was sixty-two.

Two changes were made in 1992 regarding the recognition of individuals. In previous competitions the announcements of winners alphabetically identified the top ten as the five highest ranking participants and the next five highest. The next group of 30-35 highest ranking people was designated "Honorable Mention." In 1992 the announcement of the results put the top 25 into five categories: the five highest ranking individuals, the next five highest, the next five highest, the next ten highest. Beginning in 1997 the top 25 (approximately) finishers were put into three categories: the five highest ranking individuals, the next ten highest, then the next ten highest. The number in the honorable mention group remained at about 30-35. The other change was the addition of an "Elizabeth Lowell Putnam Award" given from time to time to a female participant with a high score. Through 2012, there have been ten individual winners. Of these, Ioana Dumitriu and Alison Miller won it three times and Ana Caraiani and Melanie Wood won it twice. Dumitriu, Caraiani, and Wood were Putnam Fellows.

For most of the years between the late 1940s and the early 1990s Harvard far outpaced all others schools in the number of individuals receiving honorable mention status or higher. In 1991 Harvard had 11 and MIT had just 1 in that group. By 1993 MIT narrowed the margin to 8-6 in favor of Harvard. The first time that MIT surpassed Harvard was 1998 with the totals 11-9. In recognition of the significantly increasing number of participants, between 2002 and 2012 the number of those designed honorable mention has gradually increased from approximately 45 to 60. Since 1998 MIT has gradually widened its edge over Harvard from year to year in the number of individuals receiving honorable mention status or higher with the widest margin of 35-6 occurring in 2012. Among the ten institutions in the 2012 competition that had at least two people achieve honorable mention or higher status the number from MIT exceeded the total of the other nine Princeton (9), Harvard (6), UCLA (3), Stony Brook (3), Stanford (3), U. Michigan (3), U. Waterloo (3), Carnegie Mellon (2), and U. British Columbia (2). MIT's deep pool of talent may make it harder for it to beat Harvard in the team competition since between 1998 and 2012 Harvard has won the team competition eight times to MIT's three times. In 2012 MIT had 12 who placed in the top 25 to Harvard's 3 but only one MIT team member was in the top 25 whereas Harvard had two team members in the top 5. That put Harvard in first place and MIT in second.

**4. A PUTNAM WHO'S WHO.** Over the years many distinguished mathematicians and scientists have participated in the Putnam. Among them are Fields Medalists John Milnor, David Mumford, Daniel Quillen, Paul Cohen, and John G. Thompson (Milnor, Mumford, and Quillen were Putnam Fellows; Cohen was in the second five; Thompson received Honorable Mention). Physics Nobel Laureates who have received Honorable Mention or better are Richard Feynman, a Putnam Fellow in 1939, Kenneth G. Wilson, a two-time Putnam Fellow, Steven Weinberg, and Murray Gell-Mann. The Nobel Prize winner in Economics John Nash (of "A Beautiful Mind" fame), to his great disappointment,

finished in the second five of 147 individuals in 1947. Thompson won the $1,000,000 Abel Prize in 2008 and Milnor won it in 2011. Eric Lander, one of the principal leaders in the Human Genome Project, finished in the second five in 1976. Both Mumford and Lander are MacArthur Fellows. Distinguished computer scientist Donald Knuth received Honorable Mention in 1959. American Mathematical Society Presidents who did well in the Putnam are Irving Kaplansky (Putnam Fellow, 1938), Andrew Gleason (Putnam Fellow, 1940, 1941, 1942), Felix Browder (Putnam Fellow, 1946), and AMS and MAA President Ron Graham (Honorable Mention, 1958). Putnam Fellows in National Academy of Sciences include Elwyn Berlekamp, Felix Browder, Eugenio Calabi, Andrew Gleason, Melvin Hochster, Roger Howe, Irving Kaplansky, George W. Mackey, John W. Milnor, David Mumford, Daniel G. Quillen, Lawrence A. Shepp, Peter W. Shor, David Vogan, and Kenneth G. Wilson. Many others who have done well in the Putnam have won the prestigious research awards given by the American Mathematical Society. The 1956 Harvard team had both a future Nobel prize winner (Wilson) and a future Fields medalist (Mumford). Both were Putnam Fellows that year and Harvard's team finished first.

One might wonder how the winners of the AMS/MAA/SIAM Morgan Prize for outstanding research by an undergraduate student have done in the Putnam Competition. Of the seventeen recipients through 2012 Wood, Barton, Kane, Manolescu, and Pixton have been Putnam Fellows. Three-time Putnam Fellows Kedlaya, Ng, and Zhao received Honorable Mention for the Morgan Prize.

**5. CONCLUSION.** Table 4 provides the top five scores, the mean of the top five scores, and the median score for each competition between 1967 and 2012.[3] Note that in five of those years the median score was zero and in eight of them it was one! Between 1999 and 2012, only four times was the medium score greater than 1. Also observe that in 1995 only one point separated the highest and fifth highest scores. In the period 1967–2012 the largest gap between the top score and the fifth highest score was thirty-five, while the largest gap between highest top score and the second highest was twenty-two. The largest median in the period was 19; the average median score is 4.7; the median of the median scores is 2.5. Between 1996 and 2012 the median of the median scores is 1. The greatest number of zero scores also occurred in 2006, when 2279 out of 3640 participants registered scores of zero. The highest percentage of scores of zero occurred in 2006 with 62.6% of the scores being zero. Table 5 gives the mean score, the percentage of the score of 0, the score needed to finish in the top 200 (approximately) in the period from 1996 to 2012 and the score needed to finish in the top 500 (approximately) in the period from 1996 to 2012.

Is there a lesson to be learned by examining the results of the Putnam competition? It seems that doing well on the Putnam exam correlates well with high achievement as a professional mathematician, but many of the best research mathematicians have not scored high on the Putnam and of course many have not even taken the exam.

Oh, by the way, the cadets of West Point beat Harvard that day in 1933. A cadet had the top individual score. Army's victory was reported in the newspapers and the Army team received a special letter of congratulations from the Army Chief of Staff, General Douglas MacArthur.

Reference [6], written by Putnam Fellows Kedlaya, Poonen, and Vakil, gives the problems with solutions and commentary from the Putnam competitions from 1985-2000. References [3] and [4] are articles that relate Putnam trivia. Reference [1] is an article that provides the views of the Putnam competition by a number of Putnam fellows. The web site http://www.d.umn.edu/~jgallian/putnamfel/PF.html provides career information about Putnam Fellows. Problems and solutions from 1985-2012 are at http://amc.maa.org/a-activities/a7-problems/putnamindex.shtml

[3]This was all the data that I could locate.

Table 1. Number of participants in the first seventy-three competitions.

| *Year* | *Number* | *Year* | *Number* | *Year* | *Number* |
|---|---|---|---|---|---|
| 1938 | 163 | 1964 | 1439 | 1988 | 2096 |
| 1939 | 200 | 1965 | 1596 | 1989 | 2392 |
| 1940 | 208 | 1966 | 1526 | 1990 | 2347 |
| 1941 | 146 | 1967 | 1592 | 1991 | 2325 |
| 1942 | 114 | 1968 | 1398 | 1992 | 2421 |
| 1946 | 67 | 1969 | 1501 | 1993 | 2356 |
| 1947 | 145 | 1970 | 1445 | 1994 | 2314 |
| 1948 | 120 | 1971 | 1596 | 1995 | 2468 |
| 1949 | 155 | 1972 | 1681 | 1996 | 2407 |
| 1950 | 223 | 1973 | 2053 | 1997 | 2510 |
| 1951 | 209 | 1974 | 2159 | 1998 | 2581 |
| 1952 | 295 | 1975 | 2203 | 1999 | 2900 |
| 1953 | 256 | 1976 | 2131 | 2000 | 2818 |
| 1954 | 231 | 1977 | 2138 | 2001 | 2954 |
| 1955 | 256 | 1978 | 2019 | 2002 | 3349 |
| 1956 | 291 | 1979 | 2141 | 2003 | 2615 |
| 1957 | 377 | 1980 | 2043 | 2004 | 3733 |
| 1958 S | 430 | 1981 | 2043 | 2005 | 3545 |
| 1958 F | 506 | 1982 | 2024 | 2006 | 3640 |
| 1959 | 633 | 1983 | 2055 | 2007 | 3753 |
| 1960 | 867 | 1984 | 2149 | 2008 | 3627 |
| 1961 | 1094 | 1985 | 2079 | 2009 | 4036 |
| 1962 | 1187 | 1986 | 2094 | 2010 | 4296 |
| 1963 | 1260 | 1987 | 2170 | 2011 | 4440 |
| | | | | 2012 | 4277 |

Table 2. Number of teams 1975–2012.

| *Year* | *Number* | *Year* | *Number* | *Year* | *Number* | *Year* | *Number* |
|---|---|---|---|---|---|---|---|
| 1975 | 285 | 1985 | 264 | 1995 | 306 | 2005 | 395 |
| 1976 | 264 | 1986 | 270 | 1996 | 294 | 2006 | 402 |
| 1977 | 266 | 1987 | 277 | 1997 | 313 | 2007 | 413 |
| 1978 | 246 | 1988 | 257 | 1998 | 319 | 2008 | 405 |
| 1979 | 258 | 1989 | 288 | 1999 | 346 | 2009 | 439 |
| 1980 | 251 | 1990 | 289 | 2000 | 322 | 2010 | 442 |
| 1981 | 251 | 1991 | 291 | 2001 | 336 | 2011 | 460 |
| 1982 | 249 | 1992 | 284 | 2002 | 376 | 2012 | 402 |
| 1983 | 256 | 1993 | 291 | 2003 | 401 | | |
| 1984 | 264 | 1994 | 284 | 2004 | 411 | | |

Table 3. Winning teams in the first seventy-three competitions.

| *Institution* | *First Place* | *Second Place* | *Third Place* | *Fourth Place* | *Fifth Place* | *Putnam Fellows* |
|---|---|---|---|---|---|---|
| Harvard University | 29 | 10 | 13 | 5 | 1 | 102 |
| California Inst. Technology | 10 | 3 | 7 | 5 | 6 | 25 |
| Massachusetts Inst. Technology | 6 | 11 | 10 | 8 | 7 | 56 |
| University of Toronto | 4 | 5 | 4 | 4 | 1 | 23 |
| Washington University | 4 | 4 | | 1 | 2 | 6 |
| Duke University | 3 | 2 | 6 | | 1 | 6 |
| Brooklyn College | 3 | 1 | 1 | | | 5 |
| Michigan State University | 3 | | | 2 | | 5 |
| University of Waterloo | 2 | 3 | 6 | 2 | 5 | 8 |
| Cornell | 2 | 3 | 1 | 1 | 2 | 5 |
| Polytechnic Inst. Brooklyn | 2 | 1 | | | | 3 |
| Princeton University | 1 | 11 | 4 | 7 | 5 | 21 |
| University of Chicago | 1 | 3 | 3 | 1 | 3 | 10 |
| U. California, Berkeley | 1 | 1 | 2 | 5 | 2 | 16 |
| U. California, Davis | 1 | 1 | | 1 | | 2 |
| Queen's University | 1 | | 1 | 1 | | 1 |
| Case Western Reserve | 1 | | | 2 | 1 | 4 |
| Yale University | | 3 | 1 | 4 | 3 | 10 |
| Columbia University | | 2 | 3 | | | 8 |
| Rice University | | 1 | 1 | 1 | 1 | 3 |
| U. Pennsylvania | | 1 | 1 | 1 | | 3 |
| City College New York | | 1 | | 4 | | 10 |
| Dartmouth | | 1 | | | 1 | 2 |
| U. British Columbia | | 1 | | | 1 | 1 |
| Oberlin College | | 1 | | | | |
| Carnegie Mellon | | 1 | 2 | 1 | 1 | 3 |
| Cooper Union | | | 2 | | | 1 |
| U. California, Los Angeles | | | 2 | | 1 | 2 |
| Harvey Mudd College | | | 1 | | 1 | |
| U. Maryland, College Park | | | 1 | | 1 | |
| New York University | | | 1 | | | 3 |
| Miami University | | | 1 | | | |
| Mississippi Women's College | | | 1 | | | |
| Stanford University | | | | 6 | 2 | 3 |

Table 3 (cont.). Top 5 teams in the first seventy-three competitions.

| *Institution* | *First Place* | *Second Place* | *Third Place* | *Fourth Place* | *Fifth Place* | *Putnam Fellows* |
|---|---|---|---|---|---|---|
| U. Michigan, Ann Arbor | | | | 1 | 2 | |
| Kenyon College | | | | 1 | | 2 |
| Swarthmore | | | | 1 | | 1 |
| University of Manitoba | | | | 1 | | 1 |
| Illinois Inst. Technology | | | | 1 | | |
| McGill University | | | | 1 | | 1 |
| Stony Brook University | | | | 1 | | |
| University of Kansas | | | | | 1 | |
| U. of Minnesota Minneapolis | | | | | | 3 |
| Purdue University | | | | | | 2 |
| U. Alberta | | | | | | 2 |
| U. California, Santa Barbara | | | | | | 2 |
| U. Washington, Seattle | | | | | | 1 |

Table 4. Top five scores, mean of top 5, overall median for the 1967–2012.

| *Year* | *1* | *2* | *3* | *4* | *5* | *Top 5 mean* | *Exam median* |
|---|---|---|---|---|---|---|---|
| 1967 | 67 | 62 | 60 | 58 | 57 | 60.8 | 6 |
| 1968 | 93 | 92 | 89 | 85 | 85 | 88.8 | 10 |
| 1969 | 87 | 82 | 80 | 79 | 73 | 80.2 | 10 |
| 1970 | 116 | 107 | 104 | 97 | 96 | 104.0 | 4 |
| 1971 | 109 | 90 | 88 | 84 | 74 | 89.0 | 11 |
| 1972 | 85 | 79 | 66 | 63 | 59 | 70.4 | 4 |
| 1973 | 106 | 86 | 86 | 78 | 76 | 86.4 | 7 |
| 1974 | 77 | 70 | 62 | 61 | 57 | 65.4 | 6 |
| 1975 | 88 | 87 | 86 | 84 | 80 | 85.0 | 6 |
| 1976 | 74 | 70 | 68 | 64 | 61 | 67.4 | 2 |
| 1977 | 110 | 103 | 90 | 90 | 88 | 94.2 | 10 |
| 1978 | 90 | 77 | 74 | 73 | 71 | 77.0 | 11 |
| 1979 | 95 | 90 | 87 | 87 | 73 | 86.4 | 4 |
| 1980 | 73 | 72 | 69 | 68 | 66 | 69.6 | 3 |
| 1981 | 93 | 72 | 64 | 60 | 60 | 69.8 | 1 |
| 1982 | 98 | 90 | 88 | 85 | 82 | 88.6 | 2 |
| 1983 | 98 | 88 | 81 | 80 | 79 | 85.2 | 10 |
| 1984 | 111 | 89 | 81 | 80 | 80 | 88.2 | 10 |
| 1985 | 108 | 100 | 94 | 94 | 91 | 97.4 | 2 |
| 1986 | 90 | 89 | 86 | 82 | 81 | 85.6 | 19 |
| 1987 | 120 | 108 | 107 | 90 | 88 | 102.6 | 1 |
| 1988 | 120 | 120 | 119 | 112 | 110 | 116.2 | 16 |
| 1989 | 94 | 81 | 78 | 78 | 77 | 81.6 | 0 |
| 1990 | 93 | 92 | 87 | 77 | 77 | 85.2 | 2 |
| 1991 | 100 | 98 | 97 | 94 | 93 | 96.4 | 11 |
| 1992 | 105 | 100 | 95 | 95 | 92 | 97.4 | 2 |
| 1993 | 88 | 78 | 69 | 61 | 60 | 71.2 | 10 |
| 1994 | 102 | 101 | 99 | 88 | 87 | 95.4 | 3 |
| 1995 | 86 | 86 | 86 | 85 | 85 | 85.6 | 8 |
| 1996 | 98 | 89 | 80 | 80 | 76 | 88.4 | 2 |
| 1997 | 92 | 88 | 78 | 71 | 69 | 79.6 | 1 |
| 1998 | 108 | 106 | 103 | 100 | 98 | 103.0 | 10 |
| 1999 | 74 | 71 | 70 | 69 | 69 | 70.6 | 0 |
| 2000 | 96 | 93 | 92 | 92 | 90 | 92.6 | 0 |
| 2001 | 101 | 100 | 86 | 80 | 80 | 89.4 | 1 |
| 2002 | 116 | 108 | 106 | 96 | 96 | 104.4 | 3 |
| 2003 | 110 | 96 | 95 | 90 | 82 | 94.6 | 1 |
| 2004 | 109 | 101 | 99 | 89 | 89 | 97.4 | 0 |
| 2005 | 100 | 98 | 89 | 86 | 80 | 90.6 | 1 |
| 2006 | 101 | 99 | 98 | 92 | 92 | 96.4 | 0 |
| 2007 | 110 | 97 | 91 | 90 | 82 | 94.0 | 2 |
| 2008 | 117 | 110 | 108 | 102 | 101 | 107.4 | 1 |
| 2009 | 111 | 109 | 100 | 98 | 97 | 103.0 | 2 |
| 2010 | 120 | 118 | 117 | 110 | 109 | 114.8 | 2 |
| 2011 | 91 | 87 | 81 | 71 | 70 | 80 | 1 |
| 2012 | 100 | 87 | 81 | 80 | 78 | 85.2 | 0 |

Table 5. Mean, percent 0, Top 200 cut off, Top 500 cut off 1996–2012.

| *Year* | *Mean* | *pct. 0* | *Top 200* | *Top 500* |
|---|---|---|---|---|
| 1996 | 8.7 | 36.7 | 26 | 17 |
| 1997 | 7.3 | 47.7 | 26 | 12 |
| 1998 | 14.8 | 30.3 | 43 | 28 |
| 1999 | 6.3 | 60.2 | 22 | 11 |
| 2000 | 5.3 | 57.7 | 22 | 11 |
| 2001 | 8.9 | 44.9 | 37 | 20 |
| 2002 | 11.0 | 34.7 | 41 | 24 |
| 2003 | 7.1 | 27.8 | 31 | 18 |
| 2004 | 8.4 | 53.6 | 40 | 22 |
| 2005 | 7.9 | 46.7 | 33 | 20 |
| 2006 | 6.2 | 62.6 | 32 | 14 |
| 2007 | 7.0 | 42.5 | 31 | 21 |
| 2008 | 9.5 | 47.2 | 41 | 22 |
| 2009 | 9.5 | 43.7 | 38 | 22 |
| 2010 | 11.9 | 47.0 | 49 | 31 |
| 2011 | 4.4 | 46 | 24 | 13 |
| 2012 | 8.2 | 52.7 | 33 | 23 |

## 6. APPENDIX I: EXAMINATION QUESTIONS FOR THE FIRST WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION, APRIL 16, 1938.

MORNING SESSION: 9:00 to 12:00 NOON.

1. A solid is bounded by two bases in the horizontal planes $z = h/2$ and $z = -h/2$, and by such a surface that the area of every section in a horizontal plane is given by a formula of the sort Area $= a_0z^3 + a_1z^2 + a_2z + a_3$ (where as special cases some of the coefficients may be 0). Show that the volume is given by the formula $V = (1/6)h[B_1 + B_2 + 4M]$, where $B_1$ and $B_2$ are the areas of the bases, and $M$ is the area of the middle horizontal section. Show that the formulas for the volume of a cone and a sphere can be included in this formula when $a_0 = 0$.

2. A can buoy is to be made of three pieces, namely, a cylinder and two equal cones, the altitude of each cone being equal to the altitude of the cylinder. For a given area of surface, what shape will have the greatest volume?

3. If a particle moves in a plane, we may express its coordinates $x$ and $y$ as functions of the time $t$. If $x = t^2 - t$ and $y = t^4 + t$, show that the curve has a point of inflection at $t = 0$, and that the velocity of the moving particle has a maximum at $t = 0$.

4. A lumberman wishes to cut down a tree whose trunk is cylindrical and whose material is uniform. He will cut a notch, the two sides of which will be planes intersecting at a dihedral angle $\theta$ along a horizontal line through the axis of the cylinder. If $\theta$ is given, show that the least volume of material is cut when the plane bisecting the dihedral angle is horizontal.

5. Evaluate the limits:

(a) $\lim_{n\to\infty} \frac{n^2}{e^n}$ (b) $\lim_{x\to 0} \frac{1}{x} \int_0^x (t + \sin 2t)^{1/t} dt$

6. A swimmer stands at one corner of a square swimming pool and wishes to reach the diagonally opposite corner. If $w$ is his walking speed and $s$ is his swimming speed ($s < w$), find his path for the shortest time. [Consider two cases: (*a*) $w/s < \sqrt{2}$ and (*b*) $w/s > \sqrt{2}$].

7. TAKE EITHER (*a*) or (*b*).

(*a*) Show that the gravitational attraction exerted by a thin homogeneous spherical shell at an external point is the same as if the material of the shell were concentrated at its center.

(*b*) Determine all the straight lines which lie upon the surface $z = xy$, and draw a figure to illustrate your result.

AFTERNOON SESSION: 2:00-5:00 P.M.

8. TAKE EITHER (*a*) or (*b*).

(*a*) Let $A_{ik}$ be the cofactor of $a_{ik}$ in the determine

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}.$$

Let $D$ be the corresponding determinant with $a_{ik}$ replaced by $A_{ik}$. Prove $D = d^3$.

(*b*) Let $P(y) = Ay^2 + By + C$ be a quadratic polynomial in $y$. If the roots of the quadratic equation $P(y) - y = 0$ are $a$ and $b$ ($a \neq b$), show that $a$ and $b$ are roots of the biquadratic equation $P[P(y)] - y = 0$. Hence write down a quadratic equation which will give the other two roots, $c$ and $d$, of the biquadratic. Apply this result to solving the following biquadratic equation:

$$(y^2 - 3y + 2)^2 - 3(y^2 - 3y + 2) + 2 - y = 0.$$

9. Find all the solutions of the equation

$$yy'' - 2(y')^2 = 0$$

which pass through the point $x = 1, y = 1$.

10. A horizontal disc of diameter 3 inches is rotating at 4 revolutions per minute. A light is shining at a distant point in the plane of the disc. An insect is placed at the edge of the disc furthest from the light, facing the light. It at once starts crawling, and crawls so as always to face the light, at 1 inch per second. Set up the differential equation of motion, and find at what point the insect again reaches the edge of the disc.

11. Given the parabola $y^2 = 2mx$. What is the length of the shortest chord that is normal to the curve at one end?

12. From the center of a rectangular hyperbola a perpendicular is dropped upon a variable tangent. Find the locus of the foot of the perpendicular. Obtain the equation of the locus in polar coordinates, and sketch the curve.

13. Find the shortest distance between the plane $Ax + By + Cz + 1 = 0$ and the ellipsoid $x^2/a^2 + y^2/b^2 + x^2/c^2 = 1$. (For brevity, let

$$h = 1/\sqrt{A^2 + B^2 + C^2} \text{ and } m = \sqrt{a^2A^2 + b^2B^2 + c^2C^2}.)$$

State algebraically the condition that the plane shall lie outside the ellipsoid.

## 7. APPENDIX II: POSSIBLE MOST DIFFICULT PROBLEMS ON PUTNAM COMPETITION BETWEEN 1974-2011

1979 competition (no positive scores)

A6 Let $0 \leq p_i \leq 1$ for $i = 1, 2, \ldots, n$. Show that

$$\sum_{i=1}^{n} \frac{1}{|x - p_i|} \leq 8n(1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1})$$

for some $x$ satisfying $0 \leq x \leq 1$.

2011 competition (no positive scores)

B6 Let $p$ be an odd prime. Show that for at least $(p+1)/2$ values of $n$ in $\{0, 1, 2, \ldots, p-1\}$, $\sum_{k=0}^{p-1} k!n^k$ is not divisible by $p$.

**Acknowledgment**. The data in Table 3 was kindly provided by Jerry Heuer, Leonard Klosinski, and Jerry Alexanderson. I wish to thank Doug Jungreis, Kiran Kedlaya, Bjorn Poonen and Ravi Vakil for their comments on a draft of the article that appeared in the Monthly. No doubt this article set a record for the most number of Putnam Fellows to read a draft of a Monthly article.

# References

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