NAME: ______________________________

Problem 1. Let x, y, z be nonnegative real numbers such that

$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 2.$$

Prove that $8xyz \leq 1$.

Problem 2. Consider the polynomials

$$f(x) := x^6 + x^3 + 1, \qquad g(x) := x^2 + x + 1.$$

Denote the roots of $f(x) = 0$ by $x_1, x_2, \ldots, x_6$. Show that

$$\sum_{k=1}^{6} g(x_k) = 6.$$

Problem 3. Five points are selected in an equilateral triangle with sides of length 1. Prove that there are two of these points whose distance is less or equal to $\frac{1}{2}$.