Department of Mathematics

University of Houston

Analysis Seminar

Thursday, December 1, 2016

11:00-12:00 – Room 646 PGH

Speaker: N. Christopher Phillips (University of Oregon)

Title: Relating the mean dimension of a homeomorphism and the radius of comparison of its C*-algebra

Abstract: Let X be a compact metric space, and let $h\colon X \to X$ be a homeomorphism which is minimal, that is, there are no nontrivial h-invariant closed subsets. Interpreting h as the generator of an action of $\mathbb{Z}$ on X, from (X, h) one can construct a simple unital C*-algebra $C^*(\mathbb{Z}, X, h)$ (a special case of crossed product C*-algebras). It is the C*-algebra generated by a copy of $C(X)$ and a unitary u such that $ufu^* = f \circ h^{-1}$ for $f \in C(X)$. A general problem in operator algebras is to relate properties of h to properties of $C^*(\mathbb{Z}, X, h)$.

The *mean dimension* $\mathrm{mdim}(h)$ of a homeomorphism h is a dynamical invariant, designed so that the mean dimension of the Bernoulli shift on $([0,1]^d)^{\mathbb{Z}}$ is d. The *radius of comparison* $\mathrm{rc}(A)$ of a simple unital C*-algebra A quantifies the failure of traces on A to properly measure the "size" of positive elements of A. (If A is separable and nuclear, $\mathrm{rc}(A) = 0$ is conjecturally equivalent to classifiability in the sense of the Elliott program.)

It has been conjectured that $\mathrm{rc}(C^*(\mathbb{Z}, X, h)) = \frac{1}{2}\mathrm{mdim}(h)$ for any minimal h. In this talk, I will explain the terms above, and give some partial results towards the conjecture, including the bound $\mathrm{rc}(C^*(\mathbb{Z}, X, h)) \leq 1 + 2\,\mathrm{mdim}(h)$ for any minimal homeomorphism h.

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