EXAMPLES OF $*$-COMMUTING MAPS

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This presentation is accessible to graduate students and does not require any prior knowledge of the subject area.

Let X be a set and $S, T : X \to X$. Two functions S and T $*$-commute if $ST = TS$ and for every $(y, z) \in X \times X$ such that $S(y) = T(z)$, there exists a unique $x \in X$ such that $T(x) = y$ and $S(x) = z$. The concept of $*$-commuting maps was first introduced by Arzumanian and Renault where they studied $*$-commuting pairs of local homeomorphisms on a compact space X. Later Exel and Renault expand on this idea and provide many interesting examples. In this presentation I will discuss $*$-commuting maps in two settings: k-graphs and symbolic dynamics. This is joint work with Ben Maloney.

Higher-rank graphs (or k-graphs) were introduced by Kumjian and Pask to provide combinatorial models for the higher-rank Cuntz-Krieger C^*-algebras of Robertson and Steger. The shift maps on the infinite path space of a k-graph pairwise $*$-commute if and only if the k-graph is 1-coaligned in the sense that for each pair of paths (e, f) with the same source there exists a pair of paths (g, h) such that $ge = hf$. This equivalence is purely set-theoretic: the maps are not required to be continuous in any sense. When we restrict ourselves to a 2-graph formed from basic data, then the shifts $*$-commuting implies that the C^*-algebra of the 2-graph is simple and purely infinite.

Let A be a finite alphabet and let $A^{\mathbb{N}}$ denote the one-sided infinite sequence space of elements in A. Morphisms between shift spaces are called sliding block codes, and any such morphism τ_d is built from a block map $d : A^n \to A$. A sliding block code is a local homeomorphism precisely when the sliding block code $*$-commutes with the shift map.

Date: November 16, 2011.