Progress & Problems in Dynamics

Schedule and Abstracts

Monday, May 14th

9 — 9.30am Short introduction and orientation session.

9.30 — 10.30am Hongkun Zhang (University of Massachusetts, Amherst)

Title: SRB measures for non-equilibrium billiards
Abstract: Major statistical properties for 2-d dispersing billiards have been well understood recently, based upon fundamental work of Sinai, Bunimovich, Chernov, Young and many others. I will go over some categories of nonequilibrium billiards, which includes billiards under random or deterministic perturbations, billiards with micro-geometric structure; time-dependent billiards. If the system has a physically interesting steady state, then it belong in the class of ergodic SRB measures and their modifications. To have a better understanding of the SRB measure, we can furthermore investigate its Hausdorff dimensions, K-S entropy, as well as the projection onto lower dimensional phase space. I will also discuss some physical laws related to non-equilibrium billiards with physical origin.

10.30 — 11am Coffee break

11am — noon Peter Ashwin (University of Exeter, UK)

Title: From networks to networks; the dynamics of coupled cells
Abstract: What can one say about the connection structure of a network of coupled (dissipative) dynamical systems from observing the network dynamics? What can one say about possible (or even generic) collective dynamics of a network of coupled dynamical systems given a connection structure? These two questions and their applications have motivated various approaches to the dynamics of coupled systems in the last few decades. In the lecture I will review some theory and examples, focusing in particular on some of the dynamics that appears robustly in such systems but that is highly degenerate in less constrained systems. This includes invariant (synchrony) subspaces that are not hyperbolic, robust heteroclinic networks and cycles, cycling chaos, chaotic itinerancy, weak (Milnor) forms of attractor as well as robustly non-ergodic attractors.

noon — 2pm Lunch

2 — 3pm Mike Field (University of Houston)

Title: 21st Century problems; 20th century solutions? Averaging and Analyticity
Abstract: Technology and biology, notably neuroscience, have recently generated a wide range of new problems for dynamicists ranging from adaptivity and control in networks, hybrid dynamics (a mix of discrete, continuous and random dynamics), to asynchronous dynamics (local but not global time). At first sight, most of these problems do not seem to fit at all naturally into the natural evolution of dynamics from classical mechanics through to the modern statistical and geometric theories. Indeed, the new problems often seem to lead to an unruly tangle of disparate techniques and theory. In particular, analyticity is either absent or plays a

very limited role. In this talk we review some of the main problems, with particular reference to un-supervised learning in neuroscience and the general problem of pattern recognition ("qualitative computing"). We indicate how techniques, based on averaging, randomness and multiple time scales, have the potential to lead back to a rich feast of tractable problems and elegant theory. We will also explain the images appearing on the workshop poster.

3 — 3.30pm Coffee break

3.30 — 4.30pm Bastien Fernandez (CNRS — Aix-Marseille Université, France)

Title: Coupled map lattices beyond the uncoupled regime
Abstract: Beyond uncoupled regimes, the rigorous description of the dynamics of (piecewise) expanding coupled map lattices remains largely incomplete. To address this issue, I will consider repellers of periodic chains of linearly coupled Lorenz-type maps and present some recent results obtained by means of symbolic dynamics. In particular I will show that, while all symbolic codes are admissible for sufficiently small coupling intensity (=uncoupled regime), when the interaction strength exceeds a chain length independent threshold, a large bunch of codes is pruned and an extensive decay of the topological entropy follows suit. Moreover, this quantity appears to be continuous at the threshold and remains extensively bounded below by a positive number in a large part of the expanding regime.

4.30 — 5.30pm Mark Demers (Fairfield University)

Title: Perturbations of dispersing billiards via spectral methods
Abstract: We will discuss perturbations of the billiard map associated with a periodic Lorentz gas via the stability of the spectrum of the associated transfer operator. Recently, we constructed Banach spaces and norms on which the transfer operator for the unperturbed billiard enjoys a spectral gap. We will present a number of perturbations which fit into this functional analytic framework and for which the spectral gap persists, including: movements and deformations of scatterers, external forces with thermostatting, twists or kicks at reflections, and random perturbations composed of these various classes. This approach recovers many known results for these systems and establishes several new ones. This is joint work with Hongkun Zhang.

Tuesday, May 15th

10.30 — 11am Coffee

11am — noon James Meiss (University of Boulder)

Title: Symmetries and Invariants for Maps
Abstract: Noether's theorem shows that there is a close relation between symmetry and invariance for Hamiltonian systems; a similar relation holds (under an additional assumption) for symplectic maps. More generally, however, symmetry neither implies nor is implied by invariance. Nevertheless, the existence of a symmetry does imply that there are coordinates in which the system can be reduced to a skew-product form. I will discuss this reduction under the assumption that the symmetry has a global Poincaré section. We call this "reduction by lifting" as it involves using the symmetry flow to provide a lift of the map. This technique is related to more standard methods of reduction to the space of group orbits, but can apply when the group is neither proper nor free. However, the requirement of a global section is limiting. We will also discuss the relation between reduction by lifting and the use of Hilbert bases. We are interested

in volume-preserving dynamics where Noether's theorem does not apply. An important aspect of the symmetry/invariance equivalence is the definition of (Liouville-Arnold) integrability in the Hamiltonian case. What replaces this definition for volume preserving maps? We will discuss the notion of broad-integrability, introduced for flows by Bogoyavlenskij.

Noon — 2pm Lunch

2 — 3pm Nicolai Haydn (University of Southern California)

Title: Almost sure invariance principle for Shannon entropyi
Abstract: The Theorem of Shannon-McMillan-Breiman states that for ergodic measures the measure of cylinder sets decays exponentially at the rate of the entropy almost surely. We show that in fact for measures that are beta-mixing the information function satisfies an almost sure invariance principle. This extends previous results that established the CLT with rates of convergence.

3 — 3.30pm Coffee break

3.30 — 4.30pm Claire Postlethwaite (University of Auckland, NZ)

Title: Resonances of robust heteroclinic cycles and networks
Abstract: It is well known that heteroclinic cycles and networks can exist robustly in systems with symmetry. Resonance bifurcations are one way in which heteroclinic cycles can change stability. Such bifurcations occur when an algebraic condition on the eigenvalues of the equilibria in the cycle is satisfied, and generically are accompanied by the birth or death of a long-period periodic orbit. Although resonance bifurcations of heteroclinic cycles have been extensively studied, very little is known about resonances of heteroclinic networks. In this talk, I will first review known results on resonance bifurcations of heteroclinic cycles, and then describe new work on understanding resonance bifurcations of heteroclinic networks. In a network, at least one unstable manifold is two-dimensional; I will describe a technique to account for all the trajectories on these manifolds. For one example, we find that the sub-cycles of the network undergo resonance bifurcations as might be expected if they were isolated from the network but there is an additional resonance point due to the structure of the network at which the periodic orbits bifurcating from the different sub- cycles of the network interact. I will conclude with some avenues for future work and more complicated examples that are not amenable to this technique.

4.30 — 5.30pm Ian Melbourne (University of Surrey, UK)

Title: Diffusion and anomalous diffusion in spatially extended systems: A Huygens principle for superdiffusion
Abstract: Joint work with Ashwin, Field, Nicol and Torok showed that Brownian motion occurs naturally in chaotic dynamical systems with Euclidean symmetry. This provides an explanation/prediction of hypermeander of spiral waves in planar excitable media. In this talk, I will review the above results on Brownian motion and then turn to recent work, joint with Gottwald, on the "weakly chaotic" regime. For anisotropic systems (translation symmetry only), it is expected that there will be superdiffusion in the form of a stable Levy process. (Precise results are obtained in joint work with Zweimueller.) However for isotropic systems (Euclidean symmetry), there is an unexpected dichotomy whereby odd dimensions leads to superdiffusion as anticipated but Brownian motion prevails in even dimensions. (The dimension here means the number of spatial variables.) This is related to the corresponding dichotomy in the result of

Huygens on propagation of sound.

5.30 — 7pm Poster Session

Location: next door to conference room
Presenters: Timothy Chumley (Washington University in St. Louis), Jonathan Dawes (University of Bath, UK), Haibo Ruan (Hamburg University, Germany), Filiz Tümel (University of Houston), Yiwei Zhang (University of Exeter, UK), Qiliang Wu (University of Minnesota)

Wednesday, May 16th

9.30 — 10.30am Mark Pollicott (University of Warwick, UK)

Title: Periodic orbits for Anosov systems
Abstract: This talk will describe some of the classical results, not least to provide a context for some interesting recent advances, on periodic orbits for Anosov diffeomorphisms and flows. In particular, we will concentrate on Livsic-type theorems, counting theorems and zeta functions. The aim will be to give an accessible overview.

10.30 — 11am Coffee break

11am — noon Arnd Scheel (University of Minnesota)

Title: Wavenumber selection and coherent structures
Abstract: Many spatially extended systems exhibit nearly perfect spatially periodic patterns such as hexagons or stripes. Since in an unbounded ideal system, the wavenumber of these patterns can vary in an interval, both transient and final wavenumbers are difficult to predict. We illustrate such wavenumber selection problems in two contexts. In the first example, we discuss wavenumber selection in closed systems in the wake of fronts and its relation to Liesegang patterns. We show that when one "grows" patterns, selected wavenumbers differ dramatically from standard fastest linear mode predictions. We show a number of intriguing dynamical phenomena which are at the heart of Liesegang pattern scaling laws and more generally pattern selection in self-assembly. In the second example, we discuss how defects in Turing patterns sometimes select wavenumbers — and sometimes don't. Examples are grain boundaries (that do), inhomogeneities (that don't), and boundary conditions (that do or don't).

noon — 2pm Lunch

2 — 3pm Manfred Denker (Penn State University)

Title: Iterated function systems: A general approach.
Abstract: The talk is on joint work with M. Yuri from Hokkaido University. I will discuss basic properties of iterated function systems, a family of invertible continuous maps $f:D(f) \to X$ where $D(f)$ are closed subsets of a compact metrizable space X. In particular, thermodynamic properties will be derived.

3 — 3.30pm Coffee Break

3.30 — 4.30pm Ale Jan Homburg (VU University, Netherlands)

Title: Synchronization in forced dynamical systems
Abstract: Skew product systems $(y,x) \to (g(y),f_y(x))$, arise in contexts of random dynamical systems (where g models noise), of iterated functions systems (where g is a Bernoulli shift) and

of partially hyperbolic systems (obtained for instance as perturbations from hyperbolic g and f_y equal to the identity). The base system g forces the system f_y in the fibers. I'll discuss results for such systems focusing on bifurcations and synchronization (where orbits of initial points in the same fiber are attracted towards each other)

4.30 — 5.30pm Sandro Vaienti (Université du Sud, Toulon-Var, France)

Title: Escape Rates Formulae and Metastablilty for Randomly perturbed maps
Abstract: (Joint with W Bahsoun) We provide escape rate formulae for piecewise expanding interval maps with “random holes”. Then we obtain rigorous approximations of invariant densities of randomly perturbed metastable interval maps. We show that our escape rates formulae can be used to approximate limits of invariant densities of randomly perturbed metastable systems.