Problem of the week #1

Priyank Pillai

Question 1:

To find $\sum_{k=1}^{50} k^2$

Solution:

k	$\sum_{n=1}^{k} n^2$
1	$1^2 = 1$
2	$1^2+2^2 = 1+4 = 5$
3	$1^2+2^2+3^2 = 1+4+9 = 14$
4	$1^2+2^2+3^2+4^2 = 14+16 = 30$
5	$1^2+2^2+3^2+4^2+5^2 = 30+25 = 55$
6	$1^2+2^2+3^2+4^2+5^2+6^2 = 55+36 = 91$
7	$1^2+2^2+3^2+4^2+5^5+6^2+7^2 = 91+49 = 150$

$\Rightarrow$ Sequence:

1, 5, 14, 30, 55, 91, 150, ...

First difference: 4, 9, 16, 25, 36, 49, ...

Second difference: 5, 7, 9, 11, 13, ...

Third difference: 2, 2, 2, 2, ...

$$\sum_{n=1}^{k} n^2 = f(k) = ak^3 + bk^2 + ck + d$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 & 5 \\ 27 & 9 & 3 & 1 & 14 \\ 64 & 16 & 4 & 1 & 30 \end{pmatrix} \leftarrow$$ In augmented matrix form

$a+b+c+d = 1$

$8a+4b+2c+d = 5$

$27a+9b+3c+d = 14$

$64a+16b+4c+d = 30$

$7a+3b+c = 4$

$19a+5b+c = 9$

$37a+7b+c = 16$

$12a+2b = 5$

$18a+2b = 7$

$\Rightarrow 6a = 2 \Rightarrow a = \boxed{1/3}$

$\Rightarrow b = \frac{5 - 12(1/3)}{2} = \boxed{\frac{1}{2}}$

$\Rightarrow c = 4 - 3(1/2) - 7(1/3)$

$= \frac{24-9-14}{6} = \boxed{\frac{1}{6}}$

$\Rightarrow d = 1 - \frac{1}{6} - \frac{1}{2} - \frac{1}{3}$

$= \frac{6-1-3-2}{6} = \boxed{0}$

$$\Rightarrow \sum_{n=1}^{k} n^2 = f(k) = \frac{k^3}{3} + \frac{k^2}{2} + \frac{k}{6}$$

$$= k\left(\frac{k^2}{3} + \frac{k}{2} + \frac{1}{6}\right)$$

$$= k\left(\frac{2k^2+3k+1}{6}\right)$$

$$= \frac{k}{6}(2k^2+3k+1)$$

$$= \boxed{\frac{k}{6}(2k+1)(k+1)}$$

$$\therefore \sum_{n=1}^{50} n^2 = f(50) = \frac{50(100+1)(51)}{6} = 101(25(4 \cdot 4+1))$$

$$= 101(425)$$

$$= \boxed{42925}$$

425
× 101
425
425××
42925