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① We know that since $k \equiv 1 \bmod 2$
then $k = 2t + 1$ for some $t \in \mathbb{Z}_+$. Then $k = 2t + 1 \equiv 2 \bmod 3 \Rightarrow$
$2t \equiv 4 \bmod 3 \Rightarrow t \equiv 2 \bmod 3$ thus $t = 3g + 2$ for some $g \in \mathbb{Z}_+$. Then
$k = 2t + 1 = 6g + 5$. Now $k = 6g + 5 \equiv 6g \equiv 4 \bmod 5 \Rightarrow 3g \equiv 2 \bmod 5$
$\Rightarrow g \equiv 4 \bmod 5$ then $g = 5i + 4$ for some $i \in \mathbb{Z}_+$. Now $k = 30i + 29$
$\equiv 0 \bmod 7 \Rightarrow 2i + 1 \equiv 0 \bmod 7 \Rightarrow 2i \equiv 6 \bmod 7 \Rightarrow i \equiv 3 \bmod 7$ thus
$i = 7j + 3$ for some $j \in \mathbb{Z}_+$ thus $k = 210j + 119$. For $j = 0$ $k = 119$.

$119 \equiv 1 \bmod 2$

$119 \equiv 2 \bmod 3$

$119 \equiv 3 \bmod 4$

$119 \equiv 4 \bmod 5$

$119 \equiv 5 \bmod 6$

$119 \equiv 0 \bmod 7$

Thus 119 is the smallest k that satisfies our conditions. This result follows from the chinese remainder theorem.