Section 8.4
The Binomial Distribution

A binomial experiment has the following properties:

1. The number of trials is fixed.
2. There are two outcomes of the experiment: Success, with probability p and Failure, with probability q. Note: $p + q = 1$.
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Experiments with two outcomes are called **Bernoulli trials** or **Binomial trials**.

Finding the Probability of an Event of a Binomial Experiment
In a binomial experiment in which the probability of success in any trial is p, the probability of exactly x successes in n independent trials is given by

$$P(X = x) = C(n, x)\, p^x q^{n-x}$$

X is called a **binomial random variable** and its probability distribution is called a **binomial probability distribution**.

Example 1: Let the random variable X denote the number of girls in a two-child family. If the probability of a female birth is 0.5, construct the binomial distribution associated with this experiment.

Mean, Variance and Standard Deviation of a Random Variable

If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p, and probability of failure q, then the mean E(X), variance and standard deviation of X are given by applying the following formulas:

$\mu = np$
$Var(X) = npq$
$\sigma = \sqrt{Var(X)}$

Example 2: Consider the following binomial experiment. If the probability that a marriage will end in divorce within 20 years after its start is 0.84, what is the probability that out of 6 couples just married, in the next 20 years all will be divorced? Then find the mean of this experiment.

Example 3: Consider the following binomial experiment. If 69% of the workers at a large factory bring their lunch each day, what is the probability that in a randomly selected sample of 8 workers at least 3 bring their lunch each day? Then find the standard deviation of this experiment.

Example 4: Consider the following binomial experiment. It is estimated that one-third of the general population has blood type A^+. If a sample of 9 people is selected at random, what is the probability that no more than 1 of them have blood type A^+?