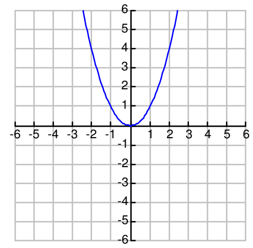
The Quadratic Function

The **quadratic function** is another parent function. The equation for the quadratic function is $y = x^2$ and its graph is a bowl-shaped curve called a **parabola**. The point $(0,0)$ is called the vertex.



The **vertex form** for all quadratics is $y = a(x-h)^2 + k$, and follows all the same rules for determining translations on the parent function except the slope. Notice the coefficient is in front of the squared term.

If $a = 1$, the parabola is standard size and 2 points are graphed up 1 and over 1 on each side of the vertex.

If $a > 1$, the parabola is skinnier which represents a vertical stretch. The graph is drawn between the basic points.

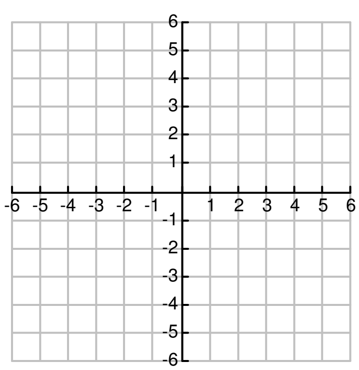
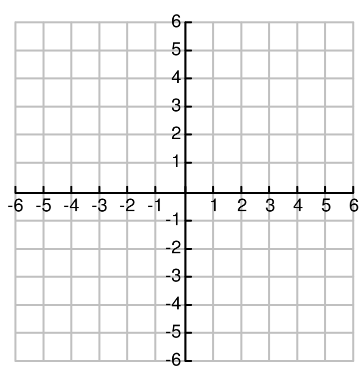
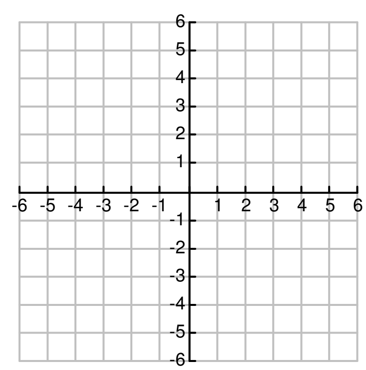
If $0 < a < 1$, the parabola is wider which represents a vertical compression. The graph is drawn outside of the basic points.

Example 1. For each problem, write the equation in the vertex form $y = a(x-h)^2 + k$.

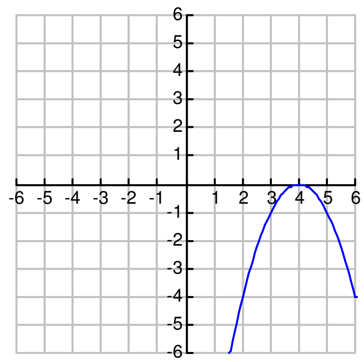
a) state the parent function
b) name the function
c) is there a reflection
d) state the vertical stretch or compression
e) state the phase (or horizontal) shift
f) state the vertical shift

a. $y = (x-2)^2$ b. $y = x^2 + 4$ c. $y = -\frac{1}{4}(x+1)^2 - 2$

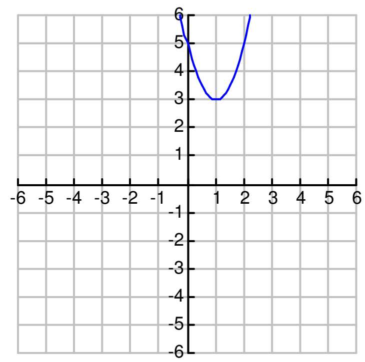
a.	b.	c.
a) __________________	a) __________________	a) __________________
b) __________________	b) __________________	b) __________________
c) __________________	c) __________________	c) __________________
d) __________________	d) __________________	d) __________________
e) __________________	e) __________________	e) __________________
f) __________________	f) __________________	f) __________________

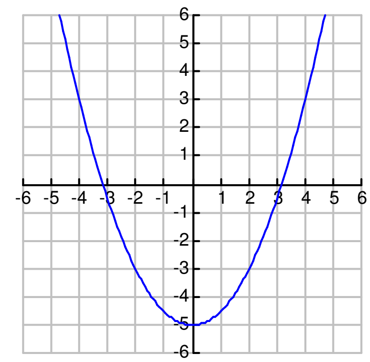
Example 2. Write the equation of each parabola from the graph and the given information.

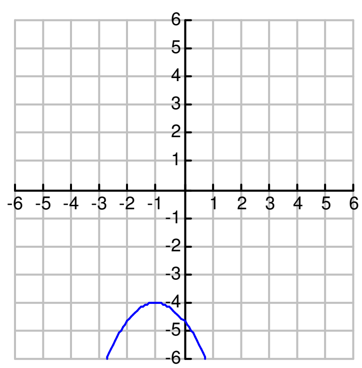


$a=2$ or $a=\frac{1}{2}$

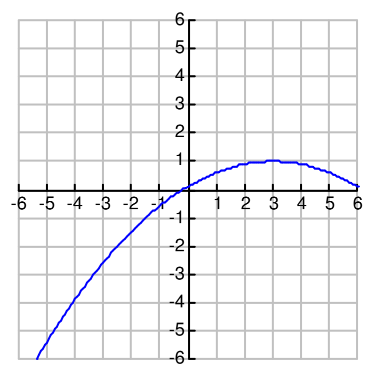


$a=\frac{2}{3}$ or $a=3$

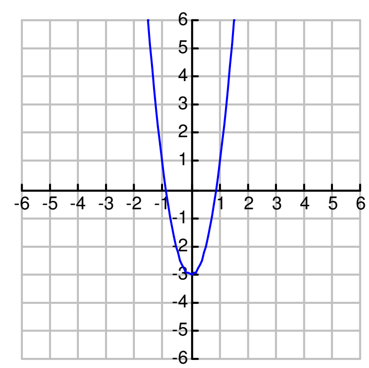




$a=4$ or $a=\frac{1}{4}$



$a=10$ or $a=\frac{1}{10}$

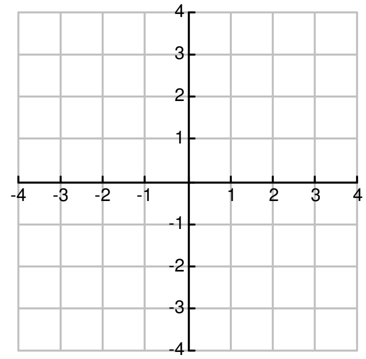


$a=.2$ or $a=5$

The Square Root Function

The **square root function** is another parent function. The equation of the square root function is $y = \sqrt{x}$.
Fill in the chart of ordered pairs and look at the graph.

x	0	1	4
y			



The graph should be a smooth curve that looks like half of a parabola.

What is the domain? ______________________

What is the range? _______________________

To determine the domain of a square root function without graphing, set the expression under the radical sign greater than or equal to zero. (The number under the square root must be 0 or a positive value.)

Example 1: Find the domain for the function $y = \sqrt{2x+3}$.
Write the answer in interval notation.

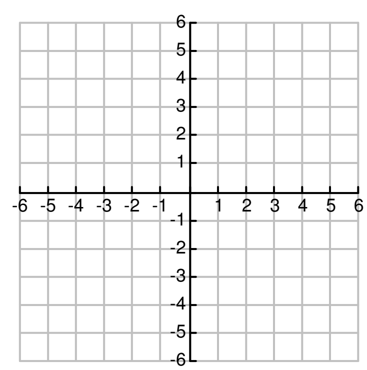
$$2x + 3 \geq 0$$
$$2x \geq -3 \qquad \text{Answer: } \left[-\frac{3}{2}, \infty\right)$$
$$x \geq -\frac{3}{2}$$

Example 2: Find the domain for the function $y = 3\sqrt{4x-5} - 1$.

The graphing form for all square root functions is $y = a\sqrt{x-h} + k$. If a < 0, the graph is reflected across the x-axis. (a flip) The **value** of **a** will determine the vertical stretch or compression. The translations are determined by h and k. Each point on the parent function moves horizontally h units and vertically k units.

Example 3: Graph $y = 3\sqrt{x+2} - 1$

Graph the parent function.
Each point on the parent function is moved horizontally to the left 2 units and vertically down 1 unit. The graph stays above the translated horizontal axis since a > 0.
Since the value of a is 3, each point on the parent function is **3 times as far** from the translated horizontal axis.

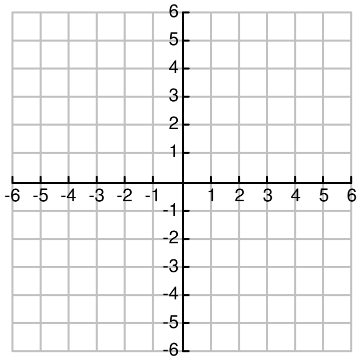


Graph the new function. State the domain and range in interval notation.

D = _________________

R = _________________

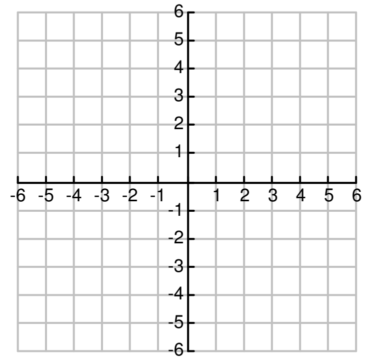
Example 4: $y = \frac{1}{2}\sqrt{x-1} + 3$



D = ____________________

R = ____________________

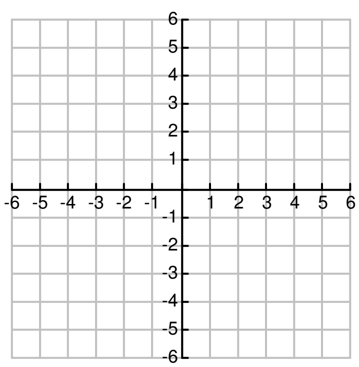
Example 5: $y = -4\sqrt{x} + 5$



D = ____________________

R = ____________________

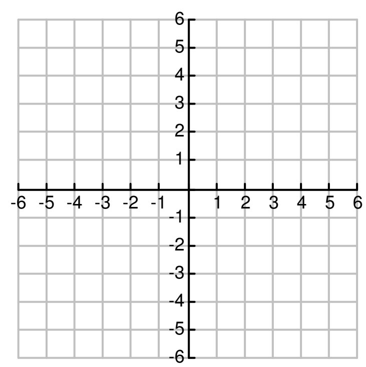
Example 6: $y = -\frac{1}{4}\sqrt{x+1}$



D = ____________________

R = ____________________

Example 7: $y = 3\sqrt{x+5} - 6$

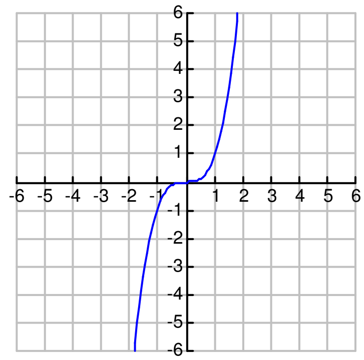


D = ____________________

R = ____________________

The Cubic Function

The **cubic function** is a parent function with the equation $y = x^3$. The graph is shown below.



The translations are performed the same way as the other functions using the equation $y = m(x - h)^3 + k$.

For each example explain the translations on the parent function to obtain the following graph.

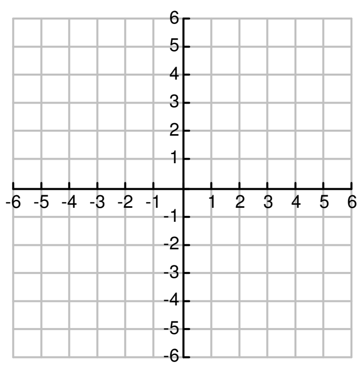
Example 1. $y = 2(x - 3)^3 + 1$

Reflection ______________________

Stretch or
compression______________________

Phase shift ______________________

Vertical shift ______________________



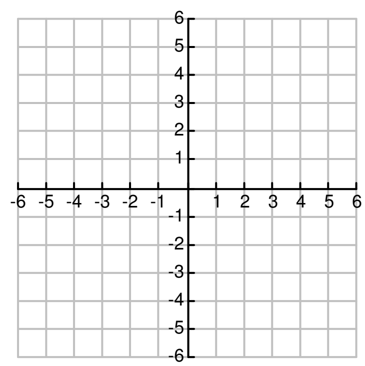
Example 2. $y = -\frac{1}{3}(x + 2)^3 - 4$

Reflection ______________________

Stretch or
compression______________________

Phase shift ______________________

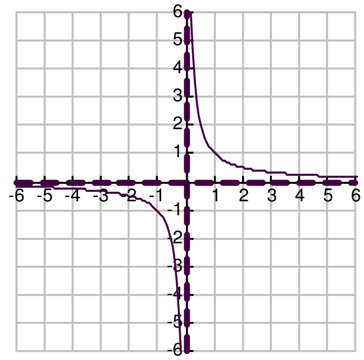
Vertical shift ______________________



The Rational Function

Another parent function is called the **rational function**. Its equation is $y = \frac{1}{x}$.

Here is its graph:



There is a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.

Instead of graphing rational functions using vertical and horizontal asymptotes, we will look at the rational functions as a family of the parent function. We will use reflections, phase shifts and vertical shifts to graph the family of rational functions.

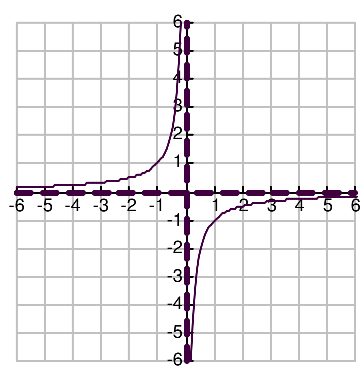
The general equation for all rational functions is:

$$y = \frac{a}{x-h} + k$$

where the sign of **a** determines a **reflection, h** determines the **phase shift** for the **vertical asymptote** and **k** determines the **vertical shift** for the **horizontal asymptote**. In this lesson we will not be concerned with finding exact values for the x and y intercepts. Our graph will be a rough sketch of the function.

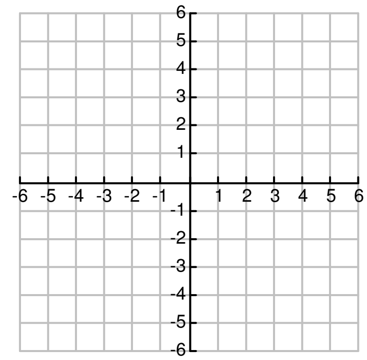
Example 1: $y = \frac{-1}{x}$

a = –1 represents a reflection (the graph starts below the x-axis, starting with the positive side)
phase shift = 0
vertical shift = 0
Graph:



Example 2 : $y = -\dfrac{1}{x+3} - 2$

Graph:



a =

phase shift =

vertical shift =

Example 3: Write the equation from the graph in the form $y = \dfrac{a}{x-h} + k$

reflection ? ________________

phase shift ________________

vertical shift ________________

Graph: