H02. Summary of tests for convergence

Test	Series	Converges	Diverges	Comment
preliminary test	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	this test cannot be used to show convergence
geometric series	$\sum_{n=0}^{\infty} ax^n$	$\|x\| < 1$	$\|x\| \geq 1$	$\sum_{n=0}^{\infty} ax^n = \frac{a}{1-x}$ $\sum_{n=0}^{k} ax^n = a\frac{1-x^{k+1}}{1-x}$
generalized harmonic series	$\sum_{n=1}^{\infty} \frac{1}{n^s}$	$s > 1$	$s \leq 1$	
comparison (a_n, $b_n \geq 0$)	$\sum_{n=1}^{\infty} a_n$	$\exists\, c: a_n \leq cb_n$ and $\sum b_n$ converges	$\exists\, d: b_n \leq da_n$ and $\sum b_n$ diverges	
integral (f is a positive decreasing function)	$\sum_{n=1}^{\infty} f(n)$	$\int_1^{\infty} f(x)dx$ is finite	$\int_1^{\infty} f(x)dx$ is infinite	
ratio ($a_n \neq 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| < 1$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| > 1$	test is inconclusive if $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = 1$
special comparison (a_n, $b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum b_n$ diverges	
alternating series ($a_n > 0$)	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	$\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$		