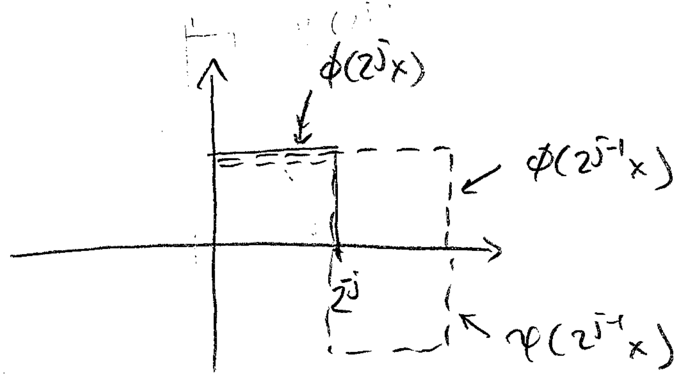
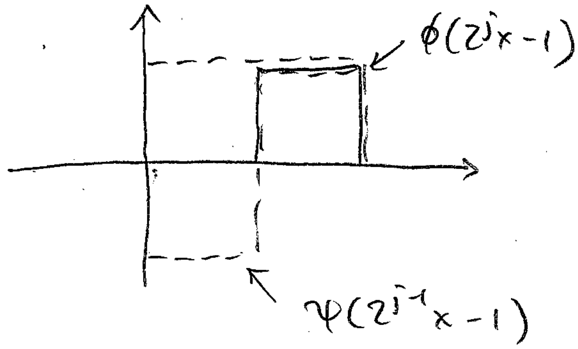
Lemma For Haar scaling fn ϕ and wavelet ψ,

$$\phi(2^j x) = \tfrac{1}{2}\left(\psi(2^{j-1}x) + \phi(2^{j-1}x)\right)$$

$$\phi(2^j x - 1) = \tfrac{1}{2}\left(\phi(2^{j-1}x) - \psi(2^{j-1}x)\right)$$





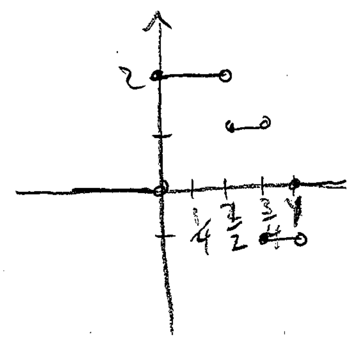
Can use this to convert $\sum c_k \phi(2^j x - k) \in V_j$

into $= \sum \left(c_k \phi(2^{j-1}x - k) + d_k \psi(2^{j-1}x - k)\right)$.

$ \qquad\qquad \uparrow V_{j-1} \qquad \uparrow W_{j-1}$

Example: From V_2 to $V_1 \oplus W_1$

$$f(x) = 2\phi(4x) + 2\phi(4x-1) + \phi(4x-2) - \phi(4x-3)$$



So $\phi(4x) = (\psi(2x) + \phi(2x))/2 \qquad (1)$

$\phi(4x-1) = (\phi(2x) - \psi(2x))/2 \qquad (2)$

$\phi(4x-2) = \phi(4(x-\tfrac{1}{2}))$

$= \cancel{\tfrac{1}{2}}(\psi(2(x-\tfrac{1}{2})) + \phi(2(x-\tfrac{1}{2})))/2 \qquad (3)$

$\phi(4x-3) = \phi(4(x-\tfrac{1}{2})-1)$

$= (\phi(2(x-\tfrac{1}{2})) - \psi(2(x-\tfrac{1}{2})))/2 \qquad (4)$

$\Rightarrow \quad f(x) = \cancel{\tfrac{1}{2}}(\psi(\cancel{2x}) - \psi(\cancel{2x})) + 2\phi(2x) \qquad 2((1)+(2))$

$+ \tfrac{1}{2}(\psi(2(x-\tfrac{1}{2})) + \psi(2(x-\tfrac{1}{2}))) \qquad (3)\mp(4)$

$+ \tfrac{1}{2}(\phi(2\cancel{(x-\tfrac{1}{2})}) - \phi(\cancel{2(x-\tfrac{1}{2})}))$

$= \underset{V_1}{\underset{\uparrow}{2\phi(2x)}} + \underset{W_1}{\underset{\uparrow}{\psi(2(x-\tfrac{1}{2}))}}$