Math 4377
Review Topics for In-Class Exam 2

1 Linear transformations

1.1 Properties

What do we need to check to verify a linear transformation is 1-1, onto, or bijective? What does it mean if it is invertible? If $T : V \to W$ and the dimension of V equals that of W, what can we say about a 1-1 transformation? What about a transformation which is onto? (Knowing rank nullity will be helpful here.)

1.2 Isomorphism

What is an isomorphism? What properties does it have (think spanning sets, linear independent sets, and bases)? If the vector space V is finite dimensional and $T : V \to W$ is an isomorphism, what about the dimension of W?

1.3 Matrix representation

How is it defined? Compare with change of coordinates. Coincidence? Know how to convert statements for the matrix representation to statements about the linear transformation. Matrix representation of the inverse? Know about similar matrices and their role for deciding whether two matrices represent the same linear transformation (with respect to different bases).

1.4 Linear functionals

What is the dual space? What is the dual of a basis? What is the annihilator? Relate dimension of subspace to that of its annihilator. Annihilator, sum and intersection of subspaces.

1.5 Double dual

How can we map a vector space to its double dual? Is this always an isomorphism?

1.6 Transpose

Matrix representation for the transpose. Given $T : V \to W$ and the coordinate vector for a linear functional f with respect to a basis for W^*, compute coordinates of $T^t f$. Know how to relate between range and kernel of T and T^t with the annihilator.

2 Determinants

2.1 n-linear functions

Direct sums, n-linear functions as functions of matrices. Alternating n-linear functions. If δ is n-linear and alternating, what is $\delta(\alpha_1, \alpha_1 + \alpha_2, \alpha_2, \alpha_3, \ldots, \alpha_{n-1})$ for vectors $\alpha_1, \alpha_2, \ldots \alpha_{n-1}$?