Math 4377
Review Topics for Final Exam

This is a compilation of the topics for the first two in-class exams, together with a few additional topics. The new topics are in **boldface**.

1 Linear equations

1.1 Systems of linear equations

Know how to solve a linear system over a field F by forming the augmented matrix and applying row operations.

1.2 Row reduction and echelon forms

Know how to transform a matrix in the row-reduced echelon form. What is the relation between elementary matrices and elementary row operations? Does the solution set to a linear system change under elementary row operations? What are independent/free variables? How can we tell there are free variables by looking at the row-reduced echelon form?

1.3 Vector equations

How can we rewrite a linear system $Ax = b$ in vector form? Can we solve the system if b can be written as a linear combination of the column vectors of A?

1.4 Solution sets

How do the solutions to an inhomogeneous system relate to the solutions of the corresponding homogeneous one?

1.5 Matrix inverse

Know how to compute the inverse of a matrix. What is the inverse of AB in terms of A^{-1} and B^{-1}? What are the properties of the system $Ax = b$ if A is invertible? **How can you use the determinant to test whether a matrix is invertible?**

2 Vector spaces

What are the defining properties of a vector space? Know the difference between finite and infinite-dimensional vector spaces.

2.1 Subspaces

Know how to test whether a subset of a vector space is a subspace.

2.2 Spanning sets and linear independence

What is the span of a set of vectors? When are vectors linearly independent?

Does the span of the row vectors of a matrix A change under elementary row operations? What about the span of column vectors?

If $Ax = 0$ has non-trivial solutions, what can we say about the column vectors of A?

2.3 Bases and dimension

You can always extend a linearly independent set to a basis by adding appropriate vectors to it. What condition is needed when adding a vector to a linearly independent set to preserve linear independence?

Compute the dimension of $W_1 + W_2$ for subspaces W_1 and W_2. How many vectors are needed for a spanning set, how many can a linearly independent set have?

2.4 Coordinates and change of coordinates

How do we compute the change of coordinates matrix? How do we compute coordinates of a vector with respect to a new basis?

3 Linear Transformations

Tell the difference between linear and non-linear transformations. Know how to compare two linear transformations efficiently by invoking a basis.

3.1 Kernel and range, nullity and rank

What is the sum of nullity and rank? Describe the properties of a linear transformation with zero rank or with zero nullity.

3.2 One-to-one, onto and invertibility properties

What do we need to check to verify a linear transformation is 1-1, onto, or bijective? What does it mean if it is invertible? If $T : V \to W$ and the dimension of V equals that of W, what can we say about a 1-1 transformation?

What about a transformation which is onto? (Knowing rank nullity will be helpful here.)

3.3 Isomorphism

What is an isomorphism? What properties does it have (think spanning sets, linear independent sets, and bases)? If the vector space V is finite dimensional and $T : V \to W$ is an isomorphism, what about the dimension of W?

3.4 Matrix representation

How is it defined? Compare with change of coordinates. Coincidence? Know how to convert statements for the matrix representation to statements about the linear transformation. Matrix representation of the inverse? Know about similar matrices and their role for deciding whether two matrices represent the same linear transformation (with respect to different bases).

3.5 Linear functionals

What is the dual space? What is the dual of a basis? What is the annihilator? Relate dimension of subspace to that of its annihilator. Annihilator, sum and intersection of subspaces.

3.6 Double dual

How can we map a vector space to its double dual? Is this always an isomorphism?

3.7 Transpose

Matrix representation for the transpose. Given $T : V \to W$ and the coordinate vector for a linear functional f with respect to a basis for W^*, compute coordinates of $T^t f$. Know how to relate between range and kernel of T and T^t with the annihilator.

4 Eigenvalues and vectors

4.1 n-linear functions

Direct sums, n-linear functions as functions of matrices. Alternating n-linear functions. If δ is n-linear and alternating, what is $\delta(\alpha_1, \alpha_1 + \alpha_2, \alpha_2, \alpha_3, \ldots, \alpha_{n-1})$ for vectors $\alpha_1, \alpha_2, \ldots \alpha_{n-1}$?

4.2 Determinant

Know how to express the determinant by cofactor expansion and by a formula involving permutations of indices. If a matrix contains lots of zero entries, which method would you use to compute the determinant? Does the determinant change under elementary row or column operations? Can you use this to compute determinants?

4.3 Characteristic polynomial

How is it defined? What are eigenvalues?

4.4 Diagonalization

What are eigenvectors? Know how to find eigenvalues and eigenvectors. Why is it nice if a matrix has a basis of eigenvectors? Which steps do you need to follow to diagonalize a matrix? What can prevent a matrix from being diagonalizable?