MATH 4377, SECTION 33224
HOMEWORK
DUE TUESDAY, SEPTEMBER 2nd

1. Find all solutions of the following system of linear equations :

$$\begin{aligned} x + y + z &= 3 \\ x - y - z &= -1 \\ 3x + y + z &= 5 \\ -x + 3y + 3z &= 5 \end{aligned}$$

2. Find all solutions of the following system of linear equations :

$$\begin{aligned} x + y + z &= 3 \\ x - y - z &= -1 \\ 3x + y + z &= 2 \\ -x + 3y + 3z &= 1 \end{aligned}$$

3. Let A be an mxn matrix over the field F and B and C nxr matrices over F. Using the formulae for matrix addition and matrix multiplication, show that

$$A (B + C) = AB + AC$$

Math 4377, Sec. 33224, Homework Doc 9/2 Key

1. $$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & -1 & -1 \\ 3 & 1 & 1 & 5 \\ -1 & 3 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -4 \\ 0 & -2 & -2 & -4 \\ 0 & 4 & 4 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

30 pts

Solutions: $x = 1$, $y = -z$ (i.e. $y + z = 2$)

2. $$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & -1 & -1 \\ 3 & 1 & 1 & 2 \\ -1 & 3 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -4 \\ 0 & -2 & -2 & -7 \\ 0 & 4 & 4 & 4 \end{array}\right]$$

30 pts

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 7 \\ 0 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$\therefore$ No solution

3. $$[A(B+C)]_{ij} = \sum_{k=1}^{n} A_{ik}(B+C)_{kj}$$

40 pts

$$= \sum_{k=1}^{n} A_{ik}(B_{kj} + C_{kj})$$

$$= \sum_{k=1}^{n} A_{ik}B_{kj} + A_{ik}C_{kj}$$

$$= \sum_{k=1}^{n} A_{ik}B_{kj} + \sum_{k=1}^{n} A_{ik}B_{kj} = (AB)_{ij} + (AC)_{ij}$$

$$= (AB+AC)_{ij}$$