Math 4377

Advanced Linear Algebra

Fall 2008

Homework Set 10, due Tuesday, Nov 11, 1pm

Section 3.6

1 Let n be a positive integer and $\mathbb{F}$ be a field. Let W be the subspace of all vectors $(x_1, x_2, \dots x_n) \in \mathbb{F}^n$ such that $x_1 + x_2 + \cdots + x_n = 0$.

(a) Prove that W^0 consists of all linear functionals f of the form

$$f(x_1, x_2, \dots x_n) = c \sum_{j=1}^{n} x_j$$

with some $c \in \mathbb{F}$.

(b) Show that every f in the dual space W^* can be written in the form

$$f(x_1, x_2, \dots x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

where $c_1 + c_2 + \cdots + c_n = 0$ and the coefficients $\{c_j\}_{j=1}^n$ are unique.

Section 3.7

1 Let $\mathbb{F}$ be a field and let f be the linear functional on $\mathbb{F}^2$ defined by $f(x_1, x_2) = ax_1 + bx_2$ for some $a, b \in \mathbb{F}$. For each of the following operators T, let $g = T^t f$ and find $g(x_1, x_2)$:

(a) $T(x_1, x_2) = (x_1, 0)$;

(b) $T(x_1, x_2) = (-x_2, x_1)$;

(c) $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$.

6 Let $P_n(\mathbb{R})$ be the vector space of all polynomials of degree at most $n \geq 1$ over $\mathbb{R}$ and let D be the differentiation operator, $Dp(x) = p'(x)$. Find a basis for the null space of D^t.

Section 5.2

1 In each of the following, D defines a function on the vector space of 3×3 matrices where $A \in \mathbb{R}^{3\times 3}$ is interpreted as the direct sum of its column vectors. In which cases is D 3-linear? Explain why!

(a) $D(A) = A_{11} + A_{22} + A_{33}$;

(b) $D(A) = A_{11}^2 + 3A_{11}A_{22}$;

(c) $D(A) = A_{11}A_{22}A_{33}$;

(d) $D(A) + A_{13}A_{22}A_{32} + 5A_{12}A_{22}A_{32}$.