Math 4377

Advanced Linear Algebra

Fall 2008

Homework Set 11, due Tuesday, Dec 9, 1pm, in 636 PGH

Section 6.2

1 In each of the following cases, T is the linear operator on $\mathbb{R}^2$ which has the matrix representation $[T]_E = A$ with respect to the standard ordered basis E for $\mathbb{R}^2$. Moreover, U is the linear operator on $\mathbb{C}^2$ with the same matrix representation, $[U]_E = A$, with respect to the standard ordered basis E for $\mathbb{C}^2$.

(a) Given

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

find the eigenvalues for T and U. For each eigenvalue λ, find a basis for the corresponding eigenspace $\ker(T - \lambda\,\mathrm{id})$ in $\mathbb{R}^2$ and $\ker(U - \lambda \mathrm{id})$ in $\mathbb{C}^2$.

(b) Find all the eigenvalues and for each eigenvalue a basis for the corresponding eigenspace of T and U if they have the matrix representation

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} .$$

It is OK to be a bit surprised.

4 Let T be the linear operator on $\mathbb{R}^3$ which is represented by the matrix

$$[T]_E = \begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$$

with respect to the standard ordered basis E. Prove that T is diagonalizable by finding a basis $B = \{\beta_1, \beta_2, \beta_3\}$ for $\mathbb{R}^3$ which consists of eigenvectors for T.

10 Prove that T on $\mathbb{R}^2$ is diagonalizable if it has the matrix representation

$$[T]_E = \begin{pmatrix} a & b \\ b & c \end{pmatrix} ,$$

with respect to the standard ordered basis E for some $a, b, c \in \mathbb{R}$.