MATH 4377, SECTION 33224
HOMEWORK
DUE THURSDAY, SEPTEMBER 11th

1. Let $G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ (a matrix over the reals) ; show that G is invertible and find G^{-1}

(An answer without computations is not acceptable)

2. Let $A = \begin{bmatrix} 1 & 1 & 0 & 3 & 2 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 2 & -1 & 5 & 2 \\ 2 & 1 & 1 & 4 & 4 \end{bmatrix}$ (over the reals) ; find rref (A) and a 4x4 invertible matrix G such that GA = rref (A). (Show your computations)

3. If G and H are nxn matrices over some field and neither G nor H is invertible, can G+H be invertible? (Yes\No answers get little to no credit)

4. Let G be an invertible nxn matrix over the field F. Show that G^t is invertible and that $(G^t)^{-1} = (G^{-1})^t$

Homework, Math 4377, Sec. 33224, Due 9/11, Key

1.
$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

30 pts

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 1 & -1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$G^{-1} = \begin{bmatrix} -1 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

2. $$\left[\begin{array}{ccccc|cccc} 1 & 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 2 & -1 & 5 & 2 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 4 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|cccc} 1 & 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & -2 & 0 & 0 & 1 \end{array}\right]$$

30 pts

$$\sim \left[\begin{array}{ccccc|cccc} 1 & 0 & 1 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right]$$

$$\therefore \quad \mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix}$$ (G is <u>not</u> unique)

check: (not necessary)

$$GA = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 3 & 2 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 2 & -1 & 5 & 2 \\ 2 & 1 & 1 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \mathrm{rref}(A)$$

Yes!

3. ~~Yes~~! Let $G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ not invertible

$H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ not invertible

10 pts

$G + H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ invertible.

4. $G G^{-1} = G^{-1} G = I_n.$

30 pts

$\therefore (G G^{-1})^t = (G^{-1} G)^t = I_n^t$

$\therefore (G^{-1})^t G^t = G^t (G^{-1})^t = I_n$

$\therefore G^t$ is invertible & $(G^t)^{-1} = (G^{-1})^t.$