

Math 4377
Advanced Linear Algebra
Fall 2008

Homework Set 5, due Thursday, Sep 25, 1pm

Section 2.2

1 Which of the following sets of vectors $\alpha = (a_1, a_2, \dots, a_n) \in \mathbb{R}^n$ are subspaces of \mathbb{R}^n ($n \geq 3$)? If yes, show it has the properties required of a subspace; if not, find an example that shows how a property of subspaces is violated:

- (a) all α such that $a_1 \geq 0$;
- (b) all α such that $a_1 + 3a_2 = a_3$;
- (c) all α such that $a_2 = a_1^2$;
- (d) all α such that $a_1 a_2 = 0$;
- (e) all α such that α_2 is rational.

4 Let W be the set of all $(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5$ which satisfy

$$\begin{array}{rcccccc} 2x_1 & - & x_2 & + & \frac{4}{3}x_3 & - & x_4 & & & = & 0 \\ x_1 & & & & + \frac{2}{3}x_3 & & & - & x_5 & = & 0 \\ 9x_1 & - & 3x_2 & + & 6x_3 & - & 3x_4 & - & 3x_5 & = & 0. \end{array}$$

Find a finite set of vectors which spans W .

5 Let F be a field and let n be an integer with $n \geq 2$. Let V be the vector space of all $n \times n$ matrices over F . Which of the following sets of matrices A are subspaces of V ? If yes, show it has the properties required of a subspace; if not, show how a property of subspaces is violated:

- (a) all invertible A ;
- (b) all non-invertible A ;
- (c) all A such that $AB = BA$ where B is a fixed matrix in V ;
- (d) all A such that $A^2 = A$.

Section 2.3

- 2 Are the vectors $\alpha_1 = (1, 1, 2, 4)$, $\alpha_2 = (2, -1, -5, 2)$, $\alpha_3 = (1, -1, -4, 0)$, $\alpha_4 = (2, 1, 1, 6)$ linearly independent in \mathbb{R}^4 ?
- 3 Find a basis for the subspace of \mathbb{R}^4 spanned by the four vectors α_1 , α_2 , α_3 and α_4 in the preceding exercise.
- 4 Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, and $\alpha_3 = (0, -3, 2)$ form a basis for \mathbb{R}^3 . Express each of the standard basis vectors ϵ_1 , ϵ_2 and ϵ_3 as a linear combination of α_1 , α_2 , and α_3 .