Math 4377

Advanced Linear Algebra

Fall 2008

Homework Set 6, due Thursday, Oct 2, 1pm

Section 2.3

6 Let V be the vector space of all 2×2 matrices over a field F. Prove that V has dimension 4 by finding a basis for V with 4 elements.

7 Let V be the vector space of the preceding exercise. Let W_1 be the set of matrices having the form

$$\begin{pmatrix} x & -x \\ y & z \end{pmatrix}$$

and the set W_2 containing all matrices of the form

$$\begin{pmatrix} a & b \\ -a & c \end{pmatrix} .$$

(a) Prove that W_1 and W_2 are subspaces of V.

(b) Find the dimensions of W_1, W_2, $W_1 + W_2$ and $W_1 \cap W_2$.

10 Let V be a vector space over a field F. Assume that V is the span of finitely many vectors $\{\alpha_!, \alpha_2, \ldots \alpha_r\}$, $r \in \mathbb{N}$. Prove that V is finite-dimensional.

Section 2.4

1 Show that the vectors $\alpha_1 = (1, 1, 0, 0)$, $\alpha_2 = (0, 0, 1, 1)$, $\alpha_3 = (1, 0, 0, 4)$, $\alpha_4 = (0, 0, 0, 4)$ form a basis for $\mathbb{R}^4$. Find the coordinates of each of the standard basis vectors ϵ_1, ϵ_2, ϵ_3 and ϵ_4 in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

2 Find the coordinate vector of $(1, 0, 1) \in \mathbb{C}^3$ relative to the ordered basis $\{(2i, 1, 0), (2, -1, 1), (0, 1 + i, 1 - i)\}$.

7 Let V be the vector space of all polynomials from $\mathbb{R}$ to $\mathbb{R}$ of maximal degree 2, so each $f \in V$ is of the form

$$f(x) = c_0 + c_1 x + c_2 x^2 ,$$

with appropriate numbers c_0, c_1, $c_2 \in \mathbb{R}$.

For a given fixed number t, let

$$g_1(x) = 1, \quad g_2(x) = x + t \quad \text{and} \quad g_3(x) = (x + t)^2 .$$

Prove that $B = \{g_1, g_2, g_3\}$ is a basis for V and compute the coordinates of the polynomial $f(x) = c_0 + c_1 x + c_2 x^2$ in this (ordered) basis B.