Math 4377
Advanced Linear Algebra
Fall 2008

Homework Set 8, due Tuesday, Oct 28, 1pm

Section 3.3

2 Let V be a vector space over the field of complex numbers and suppose T is an isomorphism of V onto $\mathbb{C}^3$. Let α_1, α_2, α_3 and α_4 be vectors in V such that $T(\alpha_1) = (1, 0, i)$, $T(\alpha_2) = (-2, 1+i, 0)$, $T(\alpha_3) = (-1, 1, 1)$, and $T(\alpha_4) = (\sqrt{2}, i, 3)$.

(a) Is α_1 in the subspace of V spanned by α_2 and α_3? Explain why/why not.

(b) Let W_1 be the suspace spanned by the set $\{\alpha_1, \alpha_2\}$ and W_2 the subspace spanned by $\{\alpha_3, \alpha_4\}$. What is the intersection of W_1 and W_2?

(c) Find a basis for the subspace spanned by $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

3 Let W be the set of all 2×2 complex Hermitian matrices, that is,

$$W = \{A \in \mathbb{C}^{2\times 2} \text{ and } A_{i,j} = \overline{A_{j,i}}\}$$

where the bar denotes complex conjugation. With the usual matrix addition and scalar multiplication with reals, this becomes a vector space over the real numbers. Verify that the linear transformation $T : \mathbb{R}^4 \to W$ given by

$$T(a, b, c, d) = \begin{pmatrix} a+d & b+ic \\ b-ic & d-a \end{pmatrix}$$

is an isomorphism onto W.

Section 3.4

2 Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by

$$T(a, b, c) = (a + b, 2c - a).$$

(a) If B is the standard ordered basis for $\mathbb{R}^3$ and B' is the standard ordered basis for $\mathbb{R}^2$, what is the matrix representation $[T]_B^{B'}$ relative to B and B'?

(b) If $B = \{\alpha_1, \alpha_2, \alpha_3\}$ and $B' = \{\beta_1, \beta_2\}$, with

$$\alpha_1 = (1, 0, -1), \quad \alpha_2 = (1, 1, 1), \quad \alpha_3 = (1, 0, 0), \quad \beta_1 = (0, 1), \quad \beta_2 = (1, 0)$$

what is the matrix representation of T relative to B and B'?

x Let T be the linear operator on $\mathbb{R}^3$ which is represented by the matrix $A = [T]_B^B$ with respect to the basis $B = \{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$, and

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}.$$

Find a basis for the range of T and a basis for the null space of T.