Math 4377

Advanced Linear Algebra

Fall 2008

Homework Set 9, due Tuesday, Nov 4, 1pm

Section 3.5

1 In $\mathbb{R}^3$, let $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (0, 1, -2)$ and $\alpha_3 = (-1, -1, 0)$.

(a) If f is a linear functional on $\mathbb{R}^3$ such that $f(\alpha_1) = 1$, $f(\alpha_2) = -1$ and $f(\alpha_3) = 3$ and $\gamma = (a, b, c)$, find $f(\gamma)$.

(b) Describe explicitly a linear functional (in terms of all of its values $f(\gamma)$ for $\gamma = (a, b, c)$) such that $f(\alpha_1) = f(\alpha_2) = 0$ but $f(\alpha_3) \neq 0$.

2 Let $B = \{\alpha_1, \alpha_2, \alpha_3\}$ be a basis for $\mathbb{C}^3$ defined by $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$ and $\alpha_3 = (2, 2, 0)$. Find the dual basis of B.

x Let $V = P_2(\mathbb{R})$ be the vector space of polynomial functions of maximal degree 2 on $\mathbb{R}$. Consider the three linear functionals $f_1(p) = \int_{-1}^{1} p(x)dx$, $f_2(p) = \int_{-1}^{1} xp(x)dx$ and $f_3(p) = \int_{-1}^{1} x^2 p(x)dx$. Show that $\{f_1, f_2, f_3\}$ is a basis for V^* by finding a basis $B = \{p_1, p_2, p_3\}$ of V for which it is the dual.

7 Let $\alpha_1 = (1, 0, -1, 2)$, $\alpha_2 = (2, 3, 1, 1)$ and let W be the subspace of $\mathbb{R}^4$ spanned by α_1 and α_2. For which choices of c_1, c_2, c_3 and c_4 in $\mathbb{R}$ is the linear functional given by

$$f(x_1, x_2, x_3, x_4) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$$

in the annihilator of W?

11 Let W_1 and W_2 be subspaces of a finite-dimensional vector space V.

(a) Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$. Hint: show that if a vector x is in the annihilator of $W_1 + W_2$, then it is in the annihilator of both W_1 and W_2. This means $(W_1 + W_2)^0 \subset W_1^0 \cap W_2^0$. Then show that the converse of the inclusion is true, too.

(b) Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$. Hint: Show that if x is in the annihilator of $W_1 \cap W_2$, then it can be written as a sum of vectors in the annihilators of W_1 and W_2. Then show the converse.