Math 4397/6397, Fall 2009
Problem Set 3, due Thursday, Sep 17

Solutions

Problem 1. If the event A denotes "success in the first shot" and B stands for "success in the second shot", then we know from the text $P(A \cup B) = 0.9$, $P(A) = 0.8$ and $P(A \cap B) = 0.7$).

a. Now from $P(B) = P(A \cup B) - P(A) + P(A \cap B)$ we get $P(B) = 0.8$, thus $P(A \cap B) = 0.7 \neq 0.8 \times 0.8 = P(A)P(B)$ and A and B are not independent.

b. $P(B|A) = \frac{P(A \cap B)}{P(A)} = 0.7/0.8 = 0.75$. $P(B|A^c) = \frac{P(A^c \cap B)}{P(A^c)} = 0.1/0.2 = 0.5$. Here we used $P(B) = P(A \cap B) + P(A^c \cap B)$ and thus $P(A^c \cap B) = P(B) - P(A \cap B) = 0.8 - 0.7 = 0.1$.

Problem 2. Use > `x <- runif(1000,0,10)` or > `x <- 10*runif(1000)`, then take the sample > `mean(x)` and sample `var(x)` and for the sample standard deviation `sqrt(var(x))` or `sd(x)`.

a. R gives

```
> x<-runif(1000,0,10)
> mean(x)
[1] 5.043496
> var(x)
[1] 8.522593
> sd(x)
[1] 2.919348
> sd(x)/sqrt(1000)
[1] 0.09231789
```

b. The theoretical values are

$$E[X] = \int_0^{10} \frac{1}{10} x dx = 5$$

and

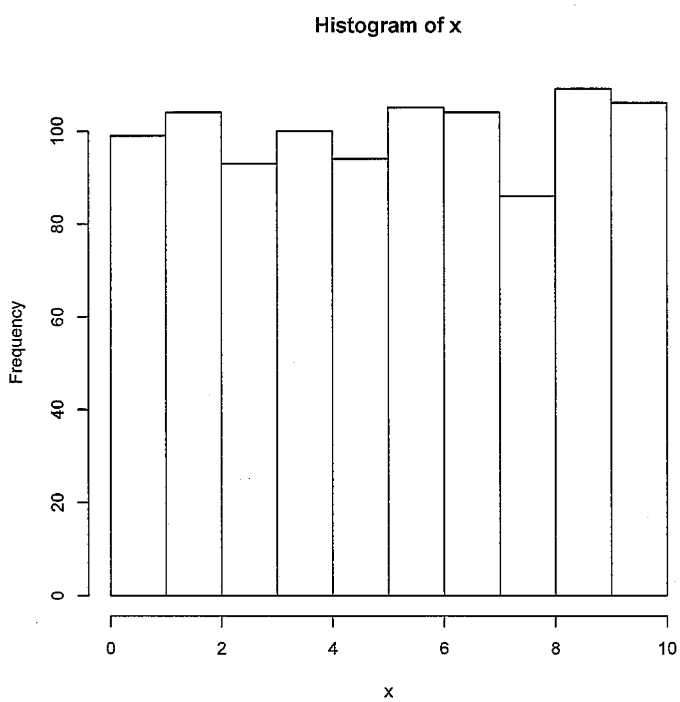
$$\mathrm{Var}(X) = E[X^2] - (E[X])^2 = \int_0^{10} \frac{1}{10} x^2 dx - 5^2 = 8\frac{1}{3}.$$

and

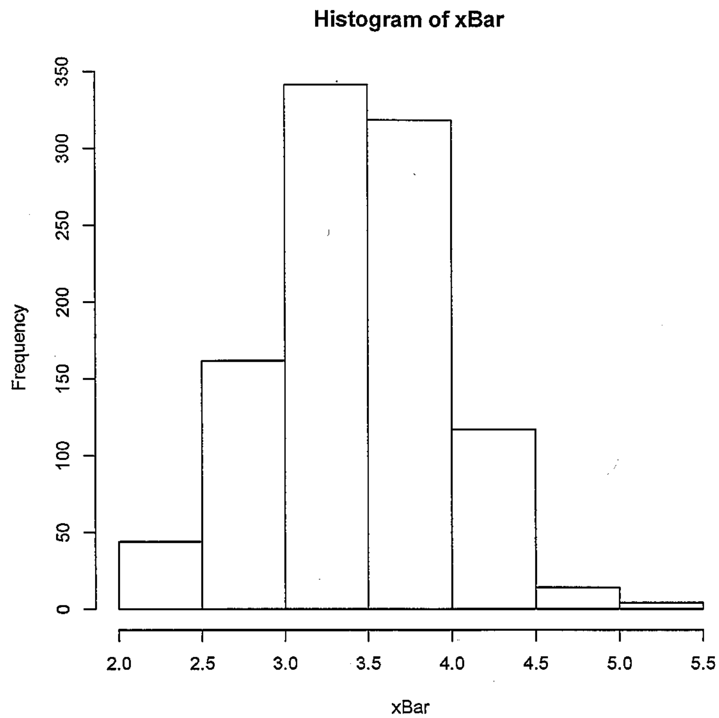
$$\text{std error of mean} = \frac{\sigma}{\sqrt{1000}} = \frac{\sqrt{25/3}}{\sqrt{1000}} \approx 0.091\,.$$

The sample mean, variance, standard deviation and std error of the mean are all close to the theoretical values obtained from the density.

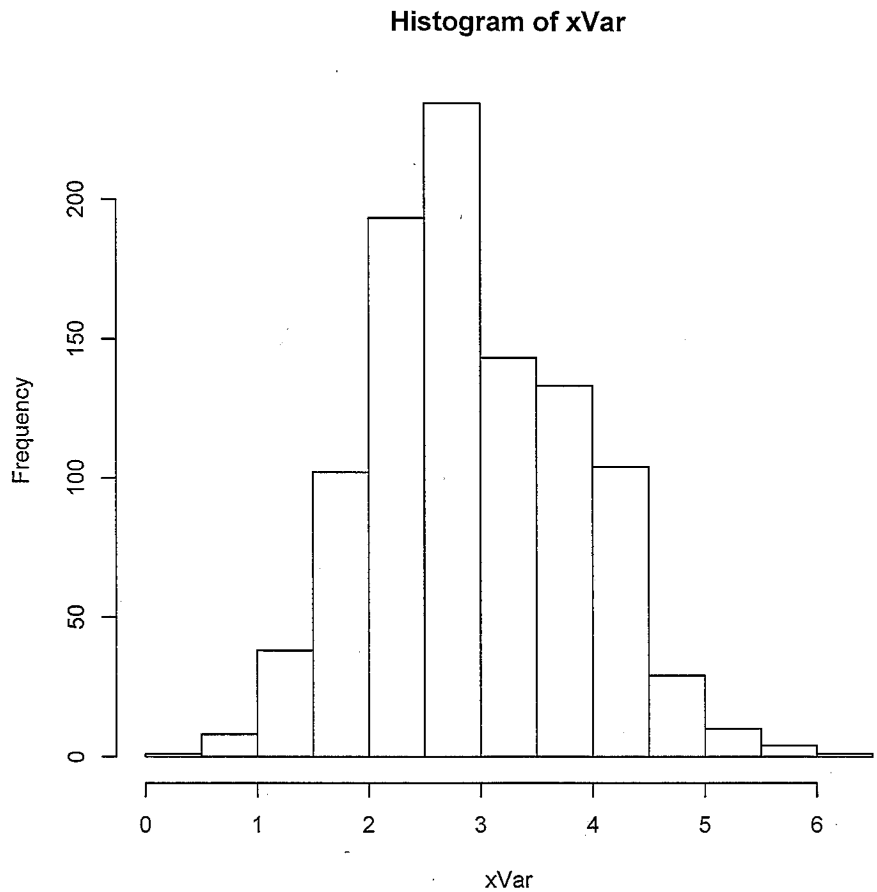
c. The histogram is obtained with `hist(x)`.



Problem 3. a. The histogram for `xBar` is obtained with `hist(xBar)`.



b. For the variance, we let `xVar<- apply(tem,1,var)` and plot `hist(xVar)`



c. The sample mean of `xBar` and `xVar` are

```
> mean(xBar)
[1] 3.4763
> var(xBar)
[1] 0.2771254
```

which means the standard deviation is

```
> sd(xBar)
[1] 0.526427
```

The sample mean should be close to the expected value. Indeed, we have

$$E[X] = \sum_{j=1}^{6} j\frac{1}{6} = \frac{(6)(7)}{2}\frac{1}{6} = 3.5$$

and

$$Var(X) = E[X^2] - (E[X])^2 = \sum_{j=1}^{6} j^2\frac{1}{6} - 3.5^2 = 91/6 - 49/4 \approx 2.91$$

and thus the standard error of the mean for averaging 10 dice is

$$\text{std error of mean } = \sigma/\sqrt{10} = \frac{\sqrt{35/12}}{\sqrt{10}} \approx 0.54\,.$$

This means, Chebyshev gives us a probability of $1 - 1/4 = 0.75$ to be within $2 \times 0.54 = 1.08$ or a probability of $1 - 1/9 \approx 0.89$ of being within $3 \times 0.54 = 1.62$ of the true mean. (The fact that we are much closer can be explained by better estimates than Chebyshev's inequality.)

d. It is also surprising that the sample standard error of the mean, `sd(xBar)` which is

```
> sd(xBar)
[1] 0.526427
```

approximates very well the (theoretical) standard error of the mean. This again could be justified by considering the known probability mass function.