Math 4397/6397, Fall 2009
Problem Set 4, due Thursday, Sep 24

Solutions

Problem 1. a. For a positive test we have

$$DLR_+ = \frac{P(+|preg)}{P(+|notpreg)} = \frac{sens}{1-spec}$$

when $spec = 0.52$, we get $DLR_+ = 1.56$. The odds of pregnancy are 1.56 times the pre-test odds. When $spec = 0.75$, the odds of pregnancy are $DLR_+ = 0.75/0.25 = 3$, three times the pretest odds.

If the test is negative, we have

$$DLR_- = \frac{P(-|preg)}{P(-|notpreg)} = \frac{1-sens}{spec}.$$

When $spec = 0.52$, $DLR_- = 0.48$. The odds after the test are roughly half of the pre-test odds. When $spec = 0.75$, $DLR_- = 0.33$. The post-test odds are 1/3 of the pre-test odds.

b. Denote the prevalence by p, then the positive predictive value

$$PV^+ = \frac{P(+|preg)P(preg)}{P(+|preg)P(preg) + P(+|notpreg)P(notpreg)} = \frac{0.75p}{0.75p + (1-0.635)(1-p)}.$$

c. For the negative predictive value,

$$PV^- = \frac{P(-|notpreg)P(notpreg)}{P(-|notpreg)P(notpreg) + P(-|preg)P(preg)} = \frac{0.635(1-p)}{0.635(1-p) + (1-0.75)p}.$$

Problem 2. (a) Using the 20 cutoff for diagnosis of Alzheimer's and the table as it appears above, we obtain the following summary of the data:

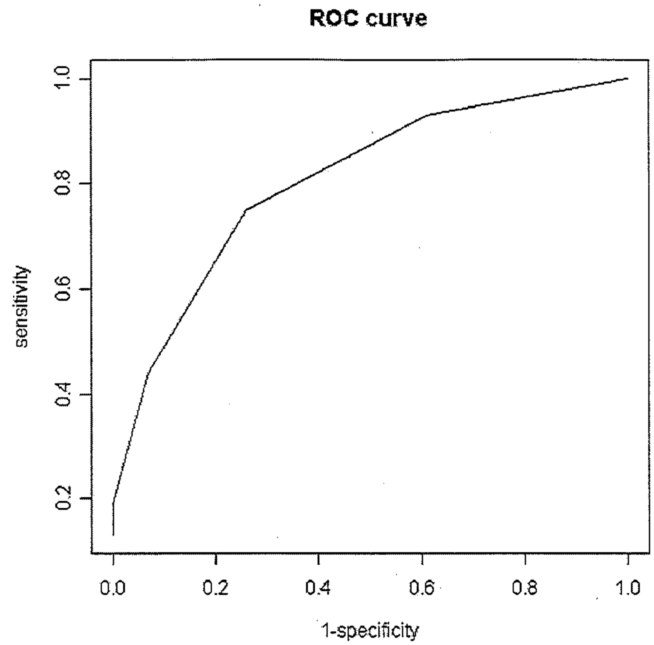
	Clinical Diagnosis	
Screen	$\overline{D}$	D
+	12	12
-	34	4
Totals	46	16

Therefore $sens = P(+ \mid D) = \frac{12}{16} = 0.75$. and $spec = P(- \mid \overline{D}) = \frac{34}{46} = 0.74$

(b) We get

Cut off	sens	1-spec
5	0.13	0
10	0.19	0
15	0.44	0.07
20	0.75	0.26
25	0.93	0.61
30	1	1

This gives the ROC curve



(c) The positive predictive value as a function of the prevalence of Alzheimer's Disease can be expressed as:

$$PV^{+} = \frac{P(+ \mid D) \times P(D)}{P(+ \mid D) \times P(D) + P(+ \mid \overline{D}) \times P(\overline{D})}$$

Using the definition of sensitivity and specificity,

$$PV^{+} = \frac{sens \times P(D)}{sens \times P(D) + (1 - spec) \times (1 - P(D))}$$

With the values from the test,

$$PV^{+} = \frac{0.75 \times P(D)}{0.75 \times P(D) + 0.26 \times (1 - P(D))}$$

The negative predictive value can be expressed as:

$$PV^{-} = \frac{P(- \mid \overline{D}) \times P(\overline{D})}{P(- \mid \overline{D}) \times P(\overline{D}) + P(- \mid D) \times P(D)}$$

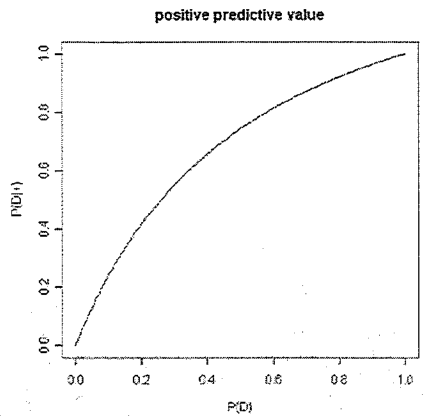
Similarly

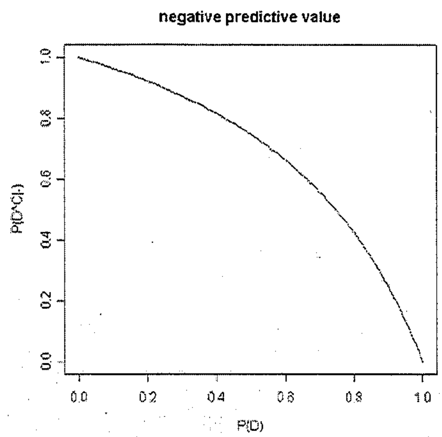
$$PV^{-} = \frac{0.74 \times (1 - P(D))}{0.74 \times (1 - P(D)) + 0.25 \times P(D)}$$

To plot the PV^{+} and PV^{-} as a function of $P(D)$, allowing the prevalence to vary from 0 to 1, we use the following code

```
sens <- .75
spec <- .74
prev <- seq(0, 1, length = 1000)
ppv <- sens * prev / (sens * prev + (1 - spec) * (1 - prev))
npv<- spec * (1-prev)/(spec * (1 - prev+(1 - sens) * prev))
plot(prev, ppv, type = "l")
plot(prev,npv,type="l")
```

See the graphs below.





Problem 3. a. We have

$$P(X = x) = \binom{10}{x} p^x (1-p)^{10-x}.$$

For a given x, the likelihood function vanishes at the boundaries, i.e. for $p = 0$ pr $p = 1$, unless $x = 0$ or $x = n$. Now let us look for critical points: The log-likelihood is

$$l(p, x) = \log(\binom{10}{x}) + x \log p + (n - x) \log(1 - p)$$

Taking the derivative with respect to p and setting to zero gives

$$\frac{dl}{dp} = \frac{x}{p} - \frac{10 - x}{1 - p} = 0.$$

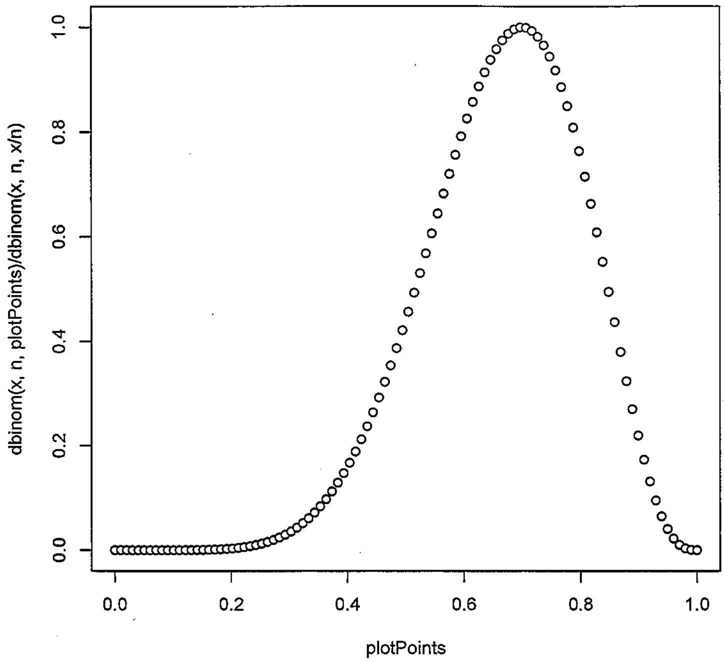
We solve for p to obtain $p = x/n$ and get a likelihood at p which is greater than zero.

In the cases $x = n$ or $x = 0$ there is no critical point inside the interval from zero to one, so we have to explicitly compare the values. We confirm that again $p = x/n$ gives the maximum.

b. For the plot, we use

```
 >plotPoints <- seq(0, 1, length = 100)
> n <- 10
> x<-7
> plot(plotPoints,dbinom(x,n,plotPoints)/dbinom(x,n,x/n))
```

which gives



Since $x = 7$, the MLE estimate for p is $7/10$. The likelihood ratio between $p = 7/10$ and $p = 5/10$ is

$$\frac{(0.7)^7(1-0.7)^3}{(0.5)^7(1-0.5)^3} = 2.27$$

so there is only weak relative evidence of a biased coin over a fair coin.

c. We compute the P-value

$$P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)\,.$$

For a fair coin, we get a probability of $P(X \geq 7) = 0.17$. So obtaining an "extreme " of this kind is not so infrequent.