Math 4397/6397
Problem Set 9, due Thu, Nov 12, 2009

Solutions

Problem 1. For non-smokers we have $\overline{x}_1 = -0.147$, $S_1 = 0.15$, $n_1 = 10$, and an unknown expected value μ_1. For smokers, we have $\overline{x}_2 = 0.053$, $S_2 = 0.225$, $n_2 = 15$ and an unknown μ_2.

We test $H_0 : \mu_1 = \mu_2$ against $H_a : \mu_1 \neq \mu_2$, which is two-sided, at $\alpha = 0.05$.

Assuming that the samples in each group are taken from i.i.d. normally distributed random variables, and that the variance is equal across the two groups, we can use the pooled variance

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(9)(0.023)+(14)(0.05)}{10+15-2} = 0.039$$

in the test statistic

$$TS = \frac{\overline{x}_1 - \overline{x}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = -2.49\,.$$

The p-value for this observation, computed with 23 degrees of freedom, is $p = 0.02 < 0.05$, so we reject the null hypothesis that there is no difference in the decline between smokers and non-smokers.

Problem 2. a. The null and alternative hypotheses are $H_0 : \mu = \mu_0 = 230$ versus $H_1 : \mu < \mu_0 = 230$. The p-value is

$$\begin{aligned} \text{p-value} &= P(\bar{x} < 220 \mid \mu = 230) \\ &= P\left(\frac{\bar{x}-230}{35/\sqrt{24}} < \frac{220-230}{\frac{35}{\sqrt{24}}} \mid \mu = 230\right) \\ &= P(t_{23} < -1.40) = 0.0874 \end{aligned}$$

Therefore the probability of observing evidence as extreme or more extreme than we got is .0874. For the typical $\alpha = .05$ we would not reject H_0.

b. The 95% confidence interval for the true mean cholesterol for people who eat the macrobiotic diet is:

$$220 \pm t_{0.975,23} \times \frac{35}{\sqrt{24}} = 220 \pm 14.78 = (205.22, 234.78)$$

Problem 3. a. To find the power, it is helpful to first find the outcomes which lead to rejection. With the significance level of the one-sided test at 0.05, we would reject the null hypothesis if $\frac{\bar{x}-230}{35/\sqrt{100}} < z_{0.05} = -1.65$. Hence we

Reject if: $\bar{x} < 230 - 1.65 \times 35/10$.

Reject if: $\bar{x} < 224.23$.

If the true mean serum cholesterol for this group is 225, the power of the test is:

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\begin{aligned}
& P(\bar{x} < 224.23 \mid \mu = 225) \\
= \; & P(Z < \frac{224.23 - 225}{35/10}) \\
= \; & P(Z < -0.22) \\
= \; & 0.4129
\end{aligned}
$$

If the true mean serum cholesterol for this group is 223, the power of the test is: $P(\bar{x} < 224.23 \mid \mu = 223) = P(Z < 0.35) = 0.6368$. If the true mean serum cholesterol for this group is 220, the power of the test is: $P(\bar{x} < 224.23 \mid \mu = 220) = P(Z < 1.21) = 0.8869$.

Problem 4. To test the equality of the variances of CA and VG, we use the following hypotheses:

$$H_0\colon \sigma^2_{CA} = \sigma^2_{VG} \quad H_1\colon \sigma^2_{CA} \neq \sigma^2_{VG}$$

Our test statistic is: $F = \frac{S^2_{CA}}{S^2_{VG}} = \frac{1.6^2}{1.4^2} = 1.306$, which gives a p-value of

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pf(1.306,35,29,lower.tail=FALSE)=.232
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so we conclude that there is no evidence in the data to suggest that the variances are different. We assume equal variances.

Problem 5. The null and alternative hypotheses are

$$H_0\colon \mu_{CA} = \mu_{VG} \quad H_1\colon \mu_{CA} \neq \mu_{VG}$$

The pooled variance is $S^2_p = \frac{35\times 1.6^2 + 29\times 1.4^2}{36+30-2} = 2.288$ and Student's t-score $= \frac{3.2-2.7}{\sqrt{2.288}\times\sqrt{1/36+1/30}} = 1.337$. The resulting p-value is 0.186 based on a t-distribution with 64 degrees of freedom, and the fact that we are testing against a two-sided alternative.

Decision: There is not sufficient evidence in the data to suggest that the mean sprint scores are different comparing toads which grew from carnivorous or vegetarian tadpoles (p-value = 0.186).

The assumptions used for this statistical test were: the scores from toads in groups CA and VG are independent, the CA scores are identically normally distributed, the VG scores are identically normally distributed, the variances of the CA and VG scores are the same.