

UNIVERSITY OF HOUSTON MIDTERM EXAMINATION

Term: Fall Year: 2009

Student: First Name _____ Last Name _____

UH Student ID Number _____

Course Abbreviation Math 4397/6397
and Number

Course Title Biostatistics

Section(s) 001

Sections of Combined
Course(s)

Section Numbers of
Combined Course(s)

Instructor(s) B. G. Bodmann

Date of Exam October 20, 2009

Time Period Start time: 2:30pm
End time: 3:50pm

Duration of Exam 1 hour 20 mins

Number of Exam 8 pages
Pages (including this
cover sheet)

Exam Type Closed book

Additional Materials Approved sheet
Allowed

Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- The last page contains **popular formulas**, including a table of selected **quantiles for the t -distribution**.
- Good luck!

1. Short answers (9 points each).

a. A well known soccer player historically scores a goal in 90% of his games. What is the probability that the player scores in two consecutive games? Explain your answer briefly stating any assumptions made.

b. Suppose that f_1 , f_2 and f_3 are all densities and let π_1, π_2, π_3 be positive numbers so that $\pi_1 + \pi_2 + \pi_3 = 1$. Show that $g(x) = \pi_1 f_1(x) + \pi_2 f_2(x) + \pi_3 f_3(x)$ is a valid density.

c. A random variable X which is exponentially distributed with rate parameter $\lambda = 3$ has the cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-3x} & , x \geq 0 \\ 0 & , \text{else} \end{cases}$$

Compute the median of X .

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2. You and your friend are playing a game where you roll a fair die. If it comes up a 1, 2, 3 or 4 he gives you a dollar. If it comes up a 5 or 6, you have to give him a dollar.
- a. (4 points) What are your expected winnings, X_1 , after the first round?
- b. (8 points) How many rounds do you have to play the game until you have a total expected winnings greater than or equal to \$5? (Show some work.)
3. (9 points) Suppose that the US intelligence quotients (IQs) are normally distributed with mean 100 and standard deviation 16. What IQ score represents the 5th percentile? (Explain your calculation.)

4. (12 points) Suppose that a sample of 100 subjects are drawn from the population of another country and that 60 of the sampled subjects have an IQ below 116. Give a 95% confidence interval estimate of the true probability p of drawing a subject from this population with an IQ below 116. State appropriate assumptions and use approximations.

5. (9 points) You are in desperate need to simulate standard normal random variables yet do not have a computer available. You do, however, have a standard six sided die. Knowing that the mean of a single die roll is 3.5 and the standard deviation is 1.71, describe how you could use the die to approximately simulate the outcomes of a standard normal random variable.

Popular formulas

1. **DeMorgan's laws** state that $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

2. **Bayes's rule**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

3. **Binomial** mass function $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, \dots, n$

4. **The normal pdf** $f(x) = (2\pi\sigma^2)^{-1/2} \exp\{-(x-\mu)^2/2\sigma^2\}$ for $-\infty < x < \infty$.

5. **Std. normal quantiles** $z_{.90}$, $z_{.95}$, $z_{.975}$ and $z_{.99}$ are 1.28, 1.645, 1.96 and 2.32, respectively.

6. **Sample variance** of data x_1, \dots, x_n with sample mean \bar{x} is

$$\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

Computing the sample variance simplifies using $\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$

7. **Variance** of a random variable X is $\sigma^2 = E[(X - \mu)^2] = E[X^2] - E[X]^2$.

8. **Chebyshev's inequality** states that for any random variable $P(|X - \mu| \geq K\sigma) \leq 1/K^2$.

9. **Central Limit Theorem.** If the X_i are iid with mean μ and (finite) variance σ^2 then

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

will be for large n distributed approximately like a standard normal random variable.

10. Assuming normality and a sample standard deviation of S , then $\left[\frac{(n-1)S^2}{\chi_{n-1, 1-\alpha/2}^2}, \frac{(n-1)S^2}{\chi_{n-1, \alpha/2}^2} \right]$ is a 100(1 - α)% **confidence interval** for σ^2 .

11. If Z is standard normal and X is independent Chi-squared with df degrees of freedom then $\frac{Z}{\sqrt{X/df}}$ follows a Gosset/Student's t distribution with df degrees of freedom.

12. For independent groups of iid variables, say X_i and Y_i , with a constant variance σ^2 across groups

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{S_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}}$$

limits to a standard normal random variable as both n_x and n_y get large. Here

$$S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$$

is the **pooled estimate** of the variance. The quantities \bar{X} , S_x , n_x are the sample mean, sample standard deviation and sample size for the X_i and \bar{Y} , S_y and n_y are defined analogously.

13. The statistic

$$\frac{\bar{Y} - \bar{X} - (\mu_y - \mu_x)}{\left(\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}\right)^{1/2}}$$

approximately follows Gosset's/Student's t distribution with degrees of freedom equal to

$$\frac{(S_x^2/n_x + S_y^2/n_y)^2}{\left(\frac{S_x^2}{n_x}\right)^2/(n_x - 1) + \left(\frac{S_y^2}{n_y}\right)^2/(n_y - 1)}$$

14. Selected **Student's t quantiles**: $t_{n,\alpha}$ where $P(T \leq t_{n,\alpha}) = \alpha$ for a random variable T which is t -distributed with n degrees of freedom

n	.9000	.9500	.9750	.9900	.9950	.9990	.9995
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659