Homework 6. Due Wednesday, October 24

Exercise 6.1. Solve **Exercise 5.3.5** from the book. You can assume that $L = \pi$. In addition,

(e) Find explicitly the zeroes of the n-th eigenfunction $\phi_n(x)$ and enumerate them in the increasing order. Namely, provide the formula for $x_i^{(n)}$ – the i-th zero of the n-th eigenfunction, $i = 1, 2, \ldots, n-1$.

(f) Let $n \geq 2$, λ_n and λ_{n+1} be two consecutive eigenvalues, and ϕ_n and ϕ_{n+1} be the corresponding eigenfunctions. Show that

$$x_i^{(n+1)} < x_i^{(n)} < x_{i+1}^{(n+1)}.$$

Exercise 6.2. Repeat the last exercise for another choice of the boundary conditions, namely, for the case $\phi(0) = 0$ and $\phi'(\pi) = 0$.

Exercise 6.3. Express in polar coordinates the 2D heat equation with constant coefficients and no sources. Consider three choices of boundary conditions.

- $\Omega_1 = \Big\{(r,\theta),\ a < r < b,\ 0 < \theta < \pi/2\Big\}$,
  $u(r,0,t) = 0F$, $u(r,\pi/2,t) = 68F$,
  $\frac{\partial u}{\partial r}(a,\theta,t) = 0F/ft$, $u(b,\theta,t) = 0F$.
- Ω_2 – exterior part of a circle of radius a, $u|_{r=a} = (50 - 5\sin(\theta))F$;
- Ω_3 – internal part of a circle of radius a, $u|_{r=a} = 50F$.

Using the symmetry reasoning, indicate which boundary conditions lead to a circularly symmetric heat flow, i.e. u does not depend on the angular variable θ.

Exercise 6.4. Read Section **5.2.2** from the book. After that, solve the Exercise **5.4.3**.