Syllabus for the
Undergraduate course MATH3363 “Introduction to PDEs”

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Textbook: R. Haberman, “Applied PDEs with Fourier series and boundary value problems”

Lecture 1.

- Partial differential equations: from physical phenomenon to analysis;
- Derivation of the conduction of heat in 1D rod; (Section 1.2, beginning).

Lecture 2.

- Conduction of heat in 1D rod: initial and boundary conditions; (Sections 1.2 and 1.3);
- Steady state solutions of 1D heat equation; (Section 1.4);

Lecture 3.

- Method of separation of variables. Main ideas;
- 1D heat equation with zero fixed temperature; (Section 2.3);

Lecture 4.

- 1D heat equation with insulated boundary conditions; (Section 2.4.1);
- 1D heat equation in a thin circular ring; (Section 2.4.2);
- Summary of boundary value problems for the 1D heat equation;

Lecture 5.

- Derivation of 2D heat equation. Divergence theorem (Gauss formula);
- Boundary conditions for 2D heat equation; (Section 1.5);
- Steady state of the 2D heat equation. Laplace equation;

Lecture 6.

- Separation of variables for the Laplace equation in a rectangle; (Section 2.5.1)

Lecture 7.

- Laplace equation in polar coordinates; (Section 2.5.2)
- Mean-value theorem. Minimum/maximum principle. Solvability (compatibility) condition for the Laplace equation; (Section 2.5.4);

Midterm test 1

Lecture 8.

- Piecewise smooth functions on a finite interval, periodic extension; (Section 3.1)
- Fourier coefficients and Fourier series. Convergence theorem. Graph sketching. (Section 3.2)

Lecture 9.

- Odd functions. Fourier coefficients of odd functions. Fourier sine series;
- Even functions. Fourier coefficients of even functions. Fourier cosine series;
- Even and odd parts of piecewise smooth functions. Alternative formulas for Fourier coefficients;

Lecture 10.

- Continuity of Fourier series, Fourier sine series, and Fourier cosine series. Graph sketching;
- Finite Fourier series. Gibbs phenomenon;

Lecture 11.

- Fundamental theorem of calculus and integration by parts for piecewise smooth functions;
- Term by term differentiation and integration of Fourier series.

Lecture 12.

- 1D wave equation. Derivation of a vertically vibrating string.
- Boundary conditions;

Lecture 13.

- 2D wave equation. Derivation of a vertically vibrating membrane.
- Boundary conditions;

Lecture 14.

- 1D wave equation. Vibrating string with fixed ends.

Lecture 15.

- Qualitative behavior of solutions of 1D heat equation; (behavior as $t \to \infty$)
- Qualitative behavior of solutions of 1D wave equation; (oscillations in time, also see the end of Section 4.4);

Midterm Test 2

Lecture 16.

- Separation of variables for heat flow in a nonuniform rod; (Section 5.2.1);
- Regular Sturm-Liouville eigenvalue problem. Physically motivated boundary conditions; (Section 5.3.2)
- Statement of theorems for a regular Sturm-Liouville problem; (Section 5.3.2)

Lecture 17.

- Illustration of theorems. Dirichlet boundary conditions; (Section 5.3.3)
- Heat flow in a nonuniform rod without sources; (Section 5.3.4)

Lecture 18.

- Rayleigh quotient: derivation.
- Rayleigh quotient and the minimization principle (pp. 189-top 191).
- Vibrations of a nonuniform string. The lower and the upper estimates of the lowest frequency of oscillation (Section 5.7);

Lecture 19.

- Review of the method of separation of variables for two variables;
- Separation of variables for 2D heat equation;
- Separation of variables for 2D wave equation;
- Cylindrical domains. Separation of variables for 3D Laplace equation;

Lecture 20.

- Statements and illustrations of theorems for the 2D Sturm-Liouville eigenvalue problem;
- Eigenvalue problem for the Laplace equation in a rectangle; Examples of simple and multiple eigenvalues.

Lecture 21. 2D Heat equation in a rectangle.

Lecture 22.

- Green formula in 2D;
- Rayleigh quotient for the diffusion operator with different choice of the boundary conditions;
- Minimal eigenvalue of the diffusion operator.

Lectures 23–25. Vibrating circular membrane and Bessel functions.

- Separation of time variable;
- Eigenvalue problem for the Laplace equation in a circle;
- Bessel equation and its solutions;
- Zeroes of the Bessel functions;
- Orthogonality of Bessel functions of the same order;
- Determination of the coefficients for a vibrating circular membrane;
- Circularly symmetric case; Case with special form of the initial conditions;
- Graphs of the Bessel functions of the first kind; Asymptotic behavior of the Bessel functions when $z \to 0$ and $z \to \infty$.